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THE DEVELOPMENT OF FREE TURBULENT SHEAR LAYERS

J. P. Lamb

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FOREWORD

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ABSTRACT

The development of a two-dimensional, free turbulent shear layer from an arbitrary initial velocity profile is analyzed theoretically. Included in the analysis are effects of both heat transfer and compressibility. The mean flow is described by approximate velocity profiles containing an unknown position parameter which is dependent upon the development length. Integral forms of the continuity and momentum equations are utilized to specify the flow characteristics along the streamline which separates the primary and secondary flow regions. By applying the Navier-Stokes equations to this dividing streamline, one is able to calculate the position parameter and thus complete the description of the developing flow field. Also presented are results of extensive calculations which show, for various external and initial flow conditions, the variation of the dividing streamline velocity, shear stress (Stanton number), and average eddy viscosity. The theory also enables one to estimate the effects of heat transfer and compressibility on the spread rate parameter for fully developed mixing zones. The theoretical results are shown to agree with experimental data from a number of sources.

CONTENTS

ILLUSTRATIONS

TABLE

NOMENCLATURE

$1/n$	Exponent in a power law velocity profile
C_∞	Free-stream Crocco number
erf	Error function
J	Dimensionless turbulent shear stress
m	Transverse turbulent normal stress
r	Longitudinal turbulent normal stress
S	Dimensionless transverse turbulent normal stress

St	Stanton number
T	Absolute temperature
U	Longitudinal free-stream velocity
U_{max}	Maximum free-stream velocity obtained by expansion to zero absolute temperature
u	Local longitudinal velocity component
v	Local transverse velocity component
X	Longitudinal reference coordinate
x	Longitudinal intrinsic coordinate
Y	Transverse reference coordinate
y	Transverse intrinsic coordinate
α	Dummy variable
β	Streamline deflection angle
δ	Initial boundary layer thickness
ϵ	Turbulent eddy viscosity
ζ	Dimensionless transverse intrinsic coordinate, y/δ
η	Homogeneous intrinsic coordinate
η_m	Coordinate displacement function
η_p	Position parameter
Λ	Local stagnation temperature ratio
λ	Overall stagnation temperature ratio
ξ	Transformed longitudinal coordinate
ρ	Fluid density
σ	Asymptotic shear layer spread rate parameter
τ	Turbulent shear stress
ϕ	Dimensionless longitudinal velocity, u/U
ψ	Dimensionless longitudinal intrinsic coordinate, x/δ
Ω	Arbitrary field variable

SUBSCRIPTS

fd	Fully developed condition
i	Origin of mixing zone

j	Dividing streamline
m	Coordinate shift due to momentum integral
min	Minimum development distance
o	Stagnation conditions
R	Upper (free-stream) edge of shear layer
ref	Reference conditions (incompressible, isoenergetic)
s	Secondary flow region
∞	Free stream

SUPERSCRIPT

*	Normalized coordinate system

SECTION I
INTRODUCTION

The theory of fully developed free turbulent shear layers, dating from the classical works of Tollmien (Ref. 1) and Görtler (Ref. 2) up through the later improvements by Korst (Ref. 3) and Crane (Ref. 4), has seen a wide spectrum of application. Such asymptotic mixing zones are characterized by a universal velocity profile and a linear growth rate from zero initial thickness. On the other hand, comparatively little progress has been made in establishing a comprehensive theory for the pre-asymptotic behavior of free turbulent layers which develop nonlinearly from a finite initial thickness. This situation, it might be noted, is in contrast to a number of successful attacks (Refs. 5, 6, and 7) on the corresponding laminar problem.

A study of developing turbulent layers is important for a number of reasons. First, there are basic differences between the developing flow and the asymptotic case which require exploration. Furthermore, it is desirable to have predictable estimates of the minimum development distance beyond which the simpler asymptotic theory can be confidently applied. And finally, investigation of nonasymptotic flows yields some quantitative information about the fully developed regime.

Some fundamental aspects of turbulent shear layer development have been previously explored in experimental studies by Chapman (Ref. 8) and Wu (Ref. 9). In the former case, incompressible jets with initial boundary layer profiles were considered, whereas the latter investigation involved the decay of a step profile in a supersonic mixing zone. Most notable of the reported theoretical treatments is that of Nash (Ref. 10), who, using an approach related to that presented here, considered the development of a boundary layer profile in incompressible, isoenergetic flow.

An approximate method, reported by Kirk (Ref. 11), deals with the development problem by matching the initial viscous layer momentum thickness to that of a hypothetical fully developed free layer which begins at some point upstream of the actual origin of mixing. It should be noted that, while such a technique may be justified on grounds of computational expediency since it gives reasonable results for nearly asymptotic shear layers, it does not yield any information about the fundamental nature of a developing flow field.

In the present report, a theory which includes the independent effects of both compressibility and heat transfer on the development of an arbitrary initial velocity profile in a steady, two-dimensional, free mixing region is presented. Although in many instances there are expansions or compressions just prior to the mixing process which affect the initial viscous layer velocity profile, these are ignored here since they have little additional effect on the shear layer development as long as the mixing occurs at quasi-constant pressure.

The geometry appropriate to the present analysis is depicted schematically in Fig. 1. The free layer grows at an increasing rate until the fully developed state is reached. Of particular interest in the present consideration will be the flow along that streamline which separates the primary and secondary flow regions. This is commonly known as the "dividing streamline" and will be denoted by the subscript "j".

SECTION II
FIRST-ORDER FLOW FIELD

Velocity profiles in the developing field are described by the method of Korst (Ref. 3), in which the usual longitudinal boundary layer equation for constant pressure flow is linearized in the Oseen sense (Ref. 12) and the kinematic viscosity is interpreted as an eddy viscosity varying only in the longitudinal direction. In dimensionless form, the resulting equation is

$$\frac{\partial \phi}{\partial \psi} = \frac{\epsilon}{U\delta} \frac{\partial^2 \phi}{\partial \zeta^2} \tag{1}$$

where the normalizing quantities are the initial boundary layer thickness, δ, and the adjacent free-stream velocity, U.

To be consistent with the classical asymptotic theory in which the eddy viscosity is $\epsilon_{fd} = xU/2\sigma^2$, one postulates the form of the average local eddy viscosity in the developing flow as $\epsilon = \epsilon_{fd} f(\psi)$ where $f(\psi)$ must approach unity in the limit. One can thus write for Eq. (1)

$$\frac{2\sigma^2}{\psi\, f(\psi)} \frac{\partial \phi}{\partial \psi} = \frac{\partial^2 \phi}{\partial \zeta^2}$$

which under the integral transformation,

$$\xi = \frac{1}{2\sigma^2} \int_0^{\psi} \psi\, f(\psi)\, d\psi \tag{2}$$

becomes the normalized heat conduction equation

$$\frac{\partial \phi}{\partial \xi} = \frac{\partial^2 \phi}{\partial \zeta^2}$$

The solution to this standard equation for an initial velocity profile given by $\phi = \phi_i(\zeta)$ and for zero secondary flow at $\zeta \to -\infty$ is

$$\phi = \frac{1}{2}\left[1 + \text{erf}(\eta - \eta_p)\right] + \frac{1}{\sqrt{\pi}} \int_{\eta-\eta_p}^{\eta} \phi_i(\tilde{a})\, e^{-a^2}\, da \tag{3}$$

in which

$$\eta = \eta_p\, \zeta \tag{4a}$$

$$\eta_p = (4\,\xi)^{-1/2} \tag{4b}$$

$$\tilde{a} = (\eta - a)/\eta_p$$

It is easily shown that, when $f(\psi)$ is unity in Eq. (2), one obtains $\xi = (\psi/2\sigma)^2$, and thus $\eta = \sigma y/x$ and $\eta_p = 0$. The velocity profile then reduces to the usual first-order asymptotic solution, $\phi = \frac{1}{2}[1 + \text{erf}(\eta)]$.

The position parameter, η_p, as Eq. (4b) reveals, is solely a function of the transformed length, ξ, and serves in Eq. (3) to distort the asymptotic profile toward the initial profile, ϕ_i. (Typical profiles are shown in Fig. 2.) Furthermore, Eq. (3) shows that the origin of mixing is represented by $\eta_p \to \infty$.

To compensate for the inherent error of the first-order profiles, the Korst theory utilizes integral equations to localize the mixing zone to the appropriate inviscid jet boundary. The relation between the reference or jet boundary coordinates (X, Y) and the intrinsic or first-order flow field coordinates (x, y) is (Fig. 3)

$$X \cong x$$

$$Y = y - y_m(x)$$

or in normalized form

$$\psi^* \cong \psi$$

$$\zeta^* = \zeta - \zeta_m(\psi)$$

Thus, in analogy with Eq. (4a), one defines

$$\eta^* = \eta_p\, \zeta^*$$

From the longitudinal momentum equation in integral form, the coordinate displacement function, $\eta_m = \zeta_m\, \eta_p$, is given by

$$\eta_m = \eta_R - \eta_p + \eta_p \int_0^1 \frac{\rho_i}{\rho_\infty} \phi_i^2\, d\zeta - \int_{-\infty}^{\infty} \frac{\rho}{\rho_\infty} \phi^2\, d\eta \tag{5}$$

in which η_R is the effective upper (free-stream) edge of the shear layer. The location, η, of any streamline within the mixing region is determined from the integral continuity equation. For the dividing streamline, η_j is given by

$$\int_{-\infty}^{\eta_j} \frac{\rho}{\rho_\infty} \phi \, d\eta = \int_{-\infty}^{\infty} \frac{\rho}{\rho_\infty} \phi (1 - \phi) \, d\eta - \eta_p \int_0^1 \frac{\rho_i}{\rho_\infty} \phi_i (1 - \phi_i) \, d\zeta \tag{6}$$

Now for a turbulent Prandtl number of unity, one may utilize Crocco's integral of the energy equation to relate temperature and velocity profiles in the mixing zone (including the initial profile). Thus

$$\Lambda = \frac{T_o}{T_{o\infty}} = \frac{T_s}{T_{o\infty}} + \left(1 - \frac{T_s}{T_{o\infty}}\right)\phi = \lambda + (1 - \lambda)\phi \tag{7}$$

in which T_s is the temperature in the secondary flow region where $\phi = 0$. One may use the above relation with the equation of state for a constant pressure ideal gas flow to obtain, for negligible transverse velocity,

$$\frac{\rho}{\rho_\infty} = \frac{1 - C_\infty^2}{\Lambda - C_\infty^2 \phi^2} \tag{8}$$

where the free-stream Crocco number, $C_\infty = U/(U_{max})$, is introduced so that the specific heat ratio does not appear in Eq. (8). Equations (5), (6), and (8) enable one to determine η_m, η_j, and ϕ_j as functions of C_∞^2, λ, ϕ_i, and η_p (see Fig. 4 for typical variations). Of these variables, all but η_p are independent.

The Korst theory thus leaves η_p dependent upon ψ through Eqs. (2) and (4b) in which the kernel, $f(\psi)$, of the integral transformation in Eq. (2) is the primary unknown. Nash (Ref. 10) begins at this point, hypothesizing a plausible form for the variation of the eddy viscosity and thereby determining η_p.

The present treatment, on the other hand, establishes the position parameter in a somewhat more rigorous manner through an application of the Navier-Stokes equations to the flow along the dividing streamline. The consideration of only a single streamline is easily justified since the first-order velocity profile at any longitudinal position is fully determined once the value of η_p for that point is known. The dividing streamline is chosen for study not only for simplicity of calculation but also because the combination of first-order profiles and integral equations is most accurate near the interface of the primary and secondary flows. The major advantage of this approach over that of Nash is that no further assumptions as to the behavior of the eddy viscosity, a purely phenomenological quantity, are necessary.

SECTION III
FLOW ALONG THE DIVIDING STREAMLINE

In order to render the governing equations in a form convenient for application to the dividing streamline flow, the relation between space derivatives in the physical (ψ^*, ζ^*) and transformed (η^*, η_p) coordinates must first be established. Recalling that η^* depends upon both ψ^* and ζ^* but η_p is a function of ψ^* only, one obtains for space derivatives of an arbitrary field variable, Ω,

$$\left(\frac{\partial \Omega}{\partial \zeta^*}\right)_j = \left(\frac{\partial \Omega}{\partial \eta^*}\right)_j \frac{\partial \eta^*}{\partial \zeta^*} = \eta_p \left(\frac{\partial \Omega}{\partial \eta^*}\right)_j$$

$$\left(\frac{\partial \Omega}{\partial \psi^*}\right)_j = \left(\frac{\partial \Omega}{\partial \eta^*}\right)_j \frac{\partial \eta^*}{\partial \eta_p} \frac{d\eta_p}{d\psi^*} + \left(\frac{\partial \Omega}{\partial \eta_p}\right)_j \frac{d\eta_p}{d\psi^*} \tag{9}$$

However, from geometric considerations, one can write

$$\left(\frac{\partial \Omega}{\partial \eta_p}\right)_j = \frac{d\Omega_j}{d\eta_p} - \left(\frac{\partial \Omega}{\partial \eta^*}\right)_j \frac{d\eta_j^*}{d\eta_p} \tag{10}$$

Now the local angle of the dividing streamline is

$$\beta_j = \left(\frac{v}{u}\right)_j = \frac{d\zeta_j^*}{d\psi^*} = \frac{d\eta_p}{d\psi^*} \frac{d}{d\eta_p}\left(\frac{\eta_j^*}{\eta_p}\right)$$

$$= \frac{1}{\eta_p} \frac{d\eta_p}{d\psi^*}\left(\frac{d\eta_j^*}{d\eta_p} - \frac{\eta_j^*}{\eta_p}\right)$$

or in more compact notation

$$\beta_j = \theta D \tag{11}$$

where $\theta = \dfrac{1}{\eta_p} \dfrac{d\eta_p}{d\psi^*}$ is a function of ψ^* alone

and $D = \dfrac{d\eta_j^*}{d\eta_p} - \dfrac{\eta_j^*}{\eta_p}$ is a function of η_p alone

The combination of Eqs. (9), (10), and (11) thus yields

$$\left(\frac{\partial \Omega}{\partial \psi^*}\right)_j = \frac{d\eta_p}{d\psi^*} \frac{d\Omega_j}{d\eta_p} - \beta_j \left(\frac{\partial \Omega}{\partial \zeta^*}\right)_j \tag{12}$$

which relates the two physical space derivatives to a transformed space derivative along the dividing streamline.

With Eq. (8) the continuity relationship

$$\frac{\partial(\rho u)}{\partial \psi^*} + \frac{\partial(\rho v)}{\partial \zeta^*} = 0 \tag{13}$$

can be written nondimensionally as

$$\frac{\partial F_1}{\partial \psi^*} + \frac{\partial(F_1 \beta)}{\partial \zeta^*} = 0$$

where $F_1 = \phi/(\Lambda - C_\infty^2 \phi^2)$. Rearranging, one obtains

$$\frac{\partial \beta}{\partial \zeta^*} = -\frac{R}{\phi}\left(\frac{\partial \phi}{\partial \psi^*} + \beta \frac{\partial \phi}{\partial \zeta^*}\right) \tag{14}$$

in which

$$R = \frac{\lambda + C_\infty^2 \phi^2}{\Lambda - C_\infty^2 \phi^2}$$

In terms of stresses, the two motion equations are

$$\rho\left(u \frac{\partial u}{\partial \psi^*} + v \frac{\partial u}{\partial \zeta^*}\right) = \frac{\partial \tau}{\partial \zeta^*} + \frac{\partial r}{\partial \psi^*}$$

$$\rho\left(u \frac{\partial v}{\partial \psi^*} + v \frac{\partial v}{\partial \zeta^*}\right) = \frac{\partial m}{\partial \zeta^*} + \frac{\partial \tau}{\partial \psi^*}$$

where r and m are normal stresses. With the usual boundary layer assumption that $\zeta^* << \psi^*$, the normal stress derivative in the longitudinal motion equation can be neglected if flow in the immediate vicinity of the origin is excluded from the analysis. Also by making use of the continuity relation, Eq. (13), the inertia terms may be rewritten so that the motion equations become

$$\frac{\partial(\rho u^2)}{\partial \psi^*} + \frac{\partial(\rho u v)}{\partial \zeta^*} = \frac{\partial \tau}{\partial \zeta^*}$$

$$\frac{\partial(\rho u v)}{\partial \psi^*} + \frac{\partial(\rho v^2)}{\partial \zeta^*} = \frac{\partial m}{\partial \zeta^*} + \frac{\partial \tau}{\partial \psi^*}$$

or in nondimensional form

$$\frac{\partial F_2}{\partial \psi^*} + \frac{\partial(F_2 \beta)}{\partial \zeta^*} = \frac{\partial J}{\partial \zeta^*}$$

$$\frac{\partial(F_2 \beta)}{\partial \psi^*} + \frac{\partial(F_2 \beta^2)}{\partial \zeta^*} = \frac{\partial S}{\partial \zeta^*} + \frac{\partial J}{\partial \psi^*}$$

in which

$$F_2 = \phi F_1$$

$$J = \tau/(\rho_\infty U^2)(1 - C_\infty^2)$$

$$S = m/(\rho_\infty U^2)(1 - C_\infty^2)$$

The relation, $dF_2 = F_1(R+1)\,d\phi$, along with Eq. (14) enables one to write the two motion equations as

$$\frac{\partial J}{\partial \zeta^*} = F_1\left(\frac{\partial \phi}{\partial \psi^*} + \beta\,\frac{\partial \phi}{\partial \zeta^*}\right) \tag{15}$$

$$\frac{\partial W}{\partial \psi^*} = \beta^2\,\frac{\partial F_2}{\partial \zeta^*} - 2\beta R\,\frac{\partial J}{\partial \zeta^*} - \frac{\partial S}{\partial \zeta^*} \tag{16}$$

where $W = J - F_2\beta$. By using the eddy viscosity concept, the transverse normal stress can be expressed as

$$m = \frac{\rho\epsilon}{\delta}\left[2\,\frac{\partial v}{\partial \zeta^*} - \frac{2}{3}\left(\frac{\partial u}{\partial \psi^*} + \frac{\partial v}{\partial \zeta^*}\right)\right]$$

But the shear stress may be approximated as $\tau \doteq \frac{\rho\epsilon}{\delta}\,\frac{\partial u}{\partial \zeta^*}$ so that, in dimensionless form, the normal stress is

$$S = \left(J\Big/\frac{\partial \phi}{\partial \zeta^*}\right)\left[2\,\frac{\partial(\phi\beta)}{\partial \zeta^*} - \frac{2}{3}\left(\frac{\partial \phi}{\partial \psi^*} + \frac{\partial(\phi\beta)}{\partial \zeta^*}\right)\right]$$

which, in view of Eq. (14), becomes

$$S = \frac{2}{3}\,J\left[(2R+1)\,Q + 2(R-1)\,\beta\right] \tag{17}$$

where

$$Q = \frac{\partial \phi}{\partial \psi^*}\Big/\frac{\partial \phi}{\partial \zeta^*} \tag{18}$$

After transverse differentiation of Eq. (17), the resulting expression can be substituted into Eq. (16) to yield

$$\left(\frac{\partial W}{\partial \psi^*}\right)_j = \left(\frac{\partial J}{\partial \zeta^*}\right)_j\left[\frac{2}{3}(2R+1)\,Q - \frac{2}{3}(R+2)\,\beta + \frac{8}{3}\,C_\infty^2\,J\right]_j + \left(\beta^2\,\frac{\partial F_2}{\partial \zeta^*}\right)_j$$

Application of Eqs. (12), (14), and (15) to the left side of the above equation enables one to write the transverse motion equation as

$$\frac{d\eta_p}{d\psi^*}\,\frac{dW_j}{d\eta_p} = \left(\frac{\partial J}{\partial \zeta^*}\right)_j\left[\frac{2}{3}(2R+1)\,Q + \frac{R-1}{3}\,\beta + \frac{8}{3}\,C_\infty^2\,J\right]_j \tag{19}$$

Now with Eqs. (12), (15), and (18), one obtains

$$\left(\frac{\partial J}{\partial \zeta^*}\right)_j = F_{1j}\left(\frac{\partial \phi}{\partial \psi^*} + \beta\,\frac{\partial \phi}{\partial \zeta^*}\right)_j = F_{1j}\,\frac{d\phi_j}{d\eta_p}\,\frac{d\eta_p}{d\psi^*} \tag{20}$$

$$Q_j = \theta\left[\frac{d\phi_j}{d\eta_p}\Big/\left(\frac{\partial \phi}{\partial \eta^*}\right)_j\right] - \beta_j \tag{21}$$

Also required in Eq. (19) is an expression for J_j which may be obtained through application of the momentum integral equation to a control volume of unit width, infinitesimal length, $d\psi^*$, and semi-infinite vertical extent $(-\infty < \zeta < \zeta_j)$. The result, analogous to the flat plate boundary layer expression, is

$$\tau_j = \frac{d}{d\psi^*} \int_{-\infty}^{\zeta_j} \rho u^2 \, d\zeta$$

or in dimensionless form

$$J_j = \frac{d}{d\psi^*} (I_{2j}/\eta_p) = \theta K \tag{22}$$

where

$$K = \frac{dI_{2j}}{d\eta_p} - \frac{I_{2j}}{\eta_p}$$

$$I_{2j} = \int_{-\infty}^{\eta_j} F_2 \, d\eta$$

Substitution of Eqs. (20), (21), and (22) into Eq. (19) yields, after division by $\eta_p \theta W_j = \eta_p \theta^2 (K - F_{2j} D)$, the final form of the transverse motion equation as

$$\frac{d}{d\eta_p} (\ln W_j) = g_1 \tag{23}$$

in which g_1, purely a function of the position parameter for specified flow conditions, is given by

$$g_1 = \frac{F_{1j}\,(d\phi_j/d\eta_p)}{K - F_{2j} D} \left[\frac{2}{3} \frac{2R_j + 1}{\left(\frac{\partial \phi}{\partial \eta^*}\right)_j} \frac{d\phi_j}{d\eta_p} - (R_j + 1) D + \frac{8}{3} C_\infty^2 K \right]$$

Typical variations of g_1 are shown in Fig. 5. As a result of the integral method utilized to specify the dividing streamline, values of the derivatives included in the expression for g_1 must be computed by numerical techniques.

SECTION IV
POSITION PARAMETER

The reduced motion equation, Eq. (23), is of second order since the quantity W_j contains the derivative $d\eta_p/d\psi^*$, and therefore two numerical integrations must be performed in order to determine the position parameter. The first integral is

$$\ln \left[W_j/(W_j)_{fd} \right] = \int_0^{\eta_p} g_1 \, d\eta_p = G_1 \tag{24}$$

Now for the asymptotic regime, Eqs. (11) and (22) can be simplified since $\frac{d}{d\eta_p} \equiv 0$ and $\eta_p = (\sigma/\psi^*) \to 0$.

Thus

$$(J_j)_{fd} = (I_{2j})_{fd}/\sigma$$

$$(\beta_j)_{fd} = (\eta^*_j)_{fd}/\sigma$$

and

$$(W_j)_{fd} = (I_{2j} - F_{2j}\,\eta^*_j)_{fd}/\sigma = N/\sigma$$

where σ is the asymptotic spread rate parameter. Equation (24) can therefore be written as

$$W_j = \theta(K - F_{2j}\,D) = (N/\sigma)\ e^{G_1} \tag{25}$$

This determines θ and thereby J_j and β_j through Eqs. (11) and (22). The position parameter itself is obtained after a second integration which yields

$$\psi^* = \int_{\infty}^{\eta_p} \left(\frac{d\eta_p}{d\psi^*}\right)^{-1} d\eta_p = \int_{\eta_p}^{\infty} (-\theta\eta_p)^{-1}\, d\eta_p\ .$$

With Eq. (25) one thus obtains

$$\psi^* = \int_{\eta_p}^{\infty} g_2\ d\eta_p \tag{26}$$

where

$$g_2 = -\sigma(K - F_{2j}\,D)\Big/N\,\eta_p\ e^{G_1}$$

Since the function g_2 varies inversely with η_p, values of the integral in Eq. (26) for very large η_p must be estimated by extrapolations of g_2. Thus one may set

$$(g_2)_{\eta_p\to\infty} \cong A_1\ e^{-A_2\eta_p}$$

where A_1 and A_2 are determined from known values of g_2.

Inasmuch as the effects of heat transfer and compressibility on the asymptotic shear layer spread rate parameter, σ, have not been completely determined (see Section V), it is also desirable to express the results of the present theory in a more universal form which is independent of σ. The parameters of interest are then $\sigma\theta$, rather than θ, and a reduced development distance ψ^*/σ. Expressions for these new quantities are obtained directly from Eqs. (25) and (26).

SECTION V
ASYMPTOTIC SHEAR LAYER GROWTH

Since the asymptotic shear layer velocity profile is purely a function of the homogeneous coordinate, $\eta = \sigma y/x$, the growth or spread rate is characterized by the parameter, σ, which is dependent on both C_∞^2 and λ. The exact nature of this dependency has not yet been completely specified despite considerable experimental investigation and theoretical postulation.

Typical of the latter are the works of Abramovich (Ref. 13), Bauer (Ref. 14), and Channapragada (Ref. 15), all of which estimate the variation of σ on the basis of some phenomenological model of free turbulence. Maydew and Reed (Ref. 16) present an excellent compilation of values of σ for isoenergetic mixing from a number of sources. The uncertainty of these results, revealed by Fig. 6, has been attributed to widely different geometries and measurement techniques.

As was the case in the determination of η_p, the present theory allows one to estimate the variation of σ without further recourse to any particular turbulence model. This is accomplished through a consideration of flow conditions along the dividing streamline as the developing layer approaches the asymptotic state. The basis for the present discussion is Eq. (25), rewritten in the form

$$\frac{\sigma W_j}{N} = e^{G_1} \tag{27}$$

Curves of the variation of $\frac{\sigma W_j}{N}$ with ψ^*/σ display the expected exponential approach to the asymptotic condition and suggest that, for $(\psi^*/\sigma) \to \infty$, Eq. (27) may be approximated by

$$\frac{\sigma W_j}{N} = 1 - A_3\, e^{-\psi^*/\sigma} \tag{28}$$

where A_3 is a constant for specified values of C_∞^2 and λ. The longitudinal derivative of Eq. (28) is therefore

$$\frac{d(\sigma W_j/N)}{d(\psi^*/\sigma)} = 1 - \frac{\sigma W_j}{N}$$

which, of course, vanishes in the asymptotic limit since $\frac{\sigma W_j}{N} \to 1$. Thus, since

$$\lim_{\psi^*/\sigma \to \infty} \left(\frac{\frac{d W_j}{d\psi^*}}{1 - \frac{\sigma W_j}{N}} \right) = \frac{\frac{d(\sigma W_j/N)}{d(\psi^*/\sigma)}}{1 - \frac{\sigma W_j}{N}} = 1$$

for all flows, one concludes that, within the limitations of the simple representation of Eq. (28), the quantity σ^2/N should also be independent of both $C_\infty{}^2$ and λ. By using incompressible isoenergetic conditions as a reference, one can then write

$$\frac{\sigma}{\sigma_{ref}} = \left(\frac{N}{N_{ref}}\right)^{1/2} \tag{29}$$

where the value of σ_{ref} has been determined experimentally by Liepman and Laufer (Ref. 17) to be 12.

The variation of σ with Crocco number for an isoenergetic jet, as found from Eq. (29), is plotted in Fig. 6 and falls slightly above the Abramovich and Bauer estimates and considerably below the Channapragada curve. While this tends to confirm the general validity of Eq. (29) for isoenergetic conditions, other values of λ yield somewhat anomalous results. For example, the value of σ for a cold jet ($\lambda>1$) is predicted to be lower and that for a hot jet ($\lambda<1$) higher than for isoenergetic flow at a given Crocco number. Furthermore, for $C_\infty{}^2 \to 1$, the curves approach different limits.

However, on purely physical grounds, one would expect a hot jet to have greater turbulence intensity than an isoenergetic jet, other things being equal, and therefore to spread somewhat faster, i. e., to have a lower value of σ, with the reverse being true for a cold jet. In addition, since the limiting condition of unit Crocco number represents a stream temperature of absolute zero, no effect of λ is expected at this point. These deviations from the expected behavior suggest that σ^2/N is not completely independent of λ as previously postulated and is, at most, independent of Crocco number for a given λ.

A more realistic estimate of the variation of σ can be easily obtained, however, by multiplying Eq. (29) by a function $K(\lambda)$ such that all curves converge to the isoenergetic value of σ/σ_{ref} at unity Crocco number. The resulting values are tabulated in Table I and plotted in Fig. 7. Recalling that the actual growth rate is inversely proportional to σ, one finds that, for incompressible flow, the hot jet ($\lambda = 0.1$) spreads 25 percent faster and the cold jet ($\lambda = 10$) about 43 percent slower than the isoenergetic jet.

Unfortunately the lack of extensive experimental growth rate data for nonisoenergetic shear layers precludes a critical assessment of the present treatment of asymptotic spread rates although the results are considered reasonable.

SECTION VI
DISCUSSION OF RESULTS

In order to illustrate the effects of the independent parameters C_∞^2, λ, and ϕ_i on the developing flow, the variations of position parameter, dividing streamline velocity, dividing streamline shear stress, and average eddy viscosity will be presented. For additional insight into the development process, it is desirable to examine the variations of the dependent variables both with and without the influence of σ. The latter, it was noted previously, can be realized through the use of the reduced development distance, ψ^*/σ, as well as the variable, $\sigma\theta$, which, with Eq. (22), leads to a reduced shear stress$^+$, σJ_j.

Shown in Fig. 8 are envelope curves of position parameter for three values of λ corresponding to hot, cold, and isoenergetic jets. When plotted in this form there is little effect of compressibility or initial profile, especially for the isoenergetic and cold jet cases. It is evident that, in many instances, a mean value of η_p for each λ would be sufficient. It is also observed that all curves converge to the value of position parameter for nearly asymptotic flow, viz., $\eta_p = \sigma/\psi^*$. The corresponding minimum development distance, ψ^*_{min}, required for the position parameter to achieve this value is presented as a function of C_∞^2, λ, and ϕ_i in Fig. 9. The value of ψ^*_{min} is seen to be proportional to λ, C_∞^2, and the power law exponent of the initial velocity profile.

The variation of position parameter with actual development distance for a 1/7-power initial velocity profile is shown in Fig. 10. These curves, which indicate an increasing effect of compressibility with jet stagnation temperature (λ^{-1}), reflect a similar influence on σ shown in Fig. 7. Also indicated in Fig. 10 are experimental values of position parameter determined by Chapman (Ref. 8) for an isoenergetic incompressible shear layer. The agreement is considered good in view of the extreme difficulty of the measurement and data reduction techniques which were required. It is also significant to note that the Nash theory (Ref. 10), as a result of its reliance on these experimental results, predicts a slightly more rapid development than the present theory.

The ratio of the local dividing streamline velocity to the corresponding fully developed value is plotted against reduced development distance in Figs. 11, 12, 13. The first set of curves (Fig. 11) illustrates the effects

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$^+$For nonisoenergetic flows with unit Prandtl number, the dividing streamline Stanton number is $St_j = (1 - C_\infty^2)\, J_j$.

of C_∞^2 and ϕ_i for isoenergetic conditions and reveals that compressibility has little influence on the development of the 1/4 profile. On the other hand, the 1/10-power profile, being more nearly like the ideal initial profile of the asymptotic theory, tends to develop more rapidly for a given Crocco number than the 1/4 power. This initial profile influence is further illustrated in Fig. 12 for constant Crocco number and isoenergetic flow. The larger values of n are representative of profiles downstream of a Prandtl-Meyer expansion. It is observed, for example, that the 1/30 profile develops to 95 percent of the asymptotic velocity in about one-fifth the distance required for the 1/4 profile. The heat transfer effect shown in Fig. 13 for a 1/7 profile is observed to be rather small, especially for the hot jet case ($\lambda = 0.1$).

The variation of dividing streamline velocity with actual development distance is depicted in Fig. 14 where the fully developed condition has been arbitrarily taken as $\eta_p = 0.01$. These curves again show the effects of compressibility and initial profile for isoenergetic flow along with the influence of heat transfer for constant Crocco number and a 1/4-power profile. The distance required to achieve $\eta_p = 0.01$ with $C_\infty^2 = 0.9$ in isoenergetic flow is observed to be three times the incompressible value. A similar ratio exists between the curves corresponding to stagnation temperature ratios of 0.1 and 10.

Shown in Figs. 15 and 16 is the variation of reduced shear stress, given as the ratio of local to asymptotic values, with ψ^*/σ. For stagnation temperature ratios of 1 and 10, this variation is characterized by an overdevelopment or overshoot, the magnitude of which is proportional to C_∞^2 and the power law exponent for a given λ. On the other hand, one finds from Fig. 16 that, as a percentage of the asymptotic value, the amount of overshoot is proportional to λ (inversely proportional to the jet stagnation temperature). It is again noted (Fig. 15) that the 1/10 profile develops more rapidly than the 1/4 profile.

The mathematical basis for this overdevelopment can be deduced from Eq. (22). This relation shows that the dimensionless shear stress is a product of two terms, one of which increases with development distance while the other decreases. Limited confirmation of this type of stress variation is found in the experimental work of Mueller et al. (Ref. 18), in which shear stresses larger than the fully developed values were measured near the origin of an incompressible isoenergetic shear layer.

The variation of actual shear stress with ψ^* is indicated in Fig. 17. It is seen that the peak stress has shifted further downstream than in Fig. 15 because of the change in σ. One also finds a significant influence of heat transfer on the value of J_j. The stress values for the hot jet (Fig. 18) are twice those for isoenergetic flow, whereas the values for the cold jet case are almost an order of magnitude smaller.

Also of interest is the variation of the average eddy viscosity across the shear layer. One can write the ratio of the local value of ϵ to the asymptotic value as

$$\frac{\epsilon}{\epsilon_{fd}} = f(\psi) = -\eta_p^{-2} \frac{\sigma\theta}{\psi^*/\sigma}$$

where $\sigma\theta$ and ψ^*/σ are known functions of η_p through Eqs. (25) and (26). It will be recalled that the original formulation of the first-order motion equation requires that the function $f(\psi)$ approach unity in the asymptotic limit. That this is indeed the case is shown in Fig. 19 where envelope curves for each λ are again indicated. The magnitude of the eddy viscosity ratio is seen to be generally proportional to λ. The similarity between Figs. 8 and 19 is readily apparent since one may show that the viscosity ratio is unity when $\eta_p = \sigma/\psi^*$.

SECTION VII
CONCLUSIONS

This study of free turbulent shear layer development leads one to conclude that:

1. The present formulation, based on the application of the Navier-Stokes equations to the dividing streamline as determined by the Korst theory, is somewhat more rigorous than the analysis of Nash and considerably more perceptive than that of Kirk.

2. The position parameter is primarily a function of the stagnation temperature ratio and almost independent of initial velocity profile for a given Crocco number. There is a minimum development distance, inversely proportional to jet stagnation temperature, beyond which the position parameter is given by $\eta_p = \sigma/\psi^*$.

3. As required in the formulation of the first-order flow field motion equations, the average eddy viscosity across the shear layer monotonically approaches an asymptotic value.

4. In general the effects of initial profile, free-stream Crocco number, and heat transfer on the development process, as predicted by the present theory, are reasonable and in agreement with available experimental data as well as purely physical reasoning.

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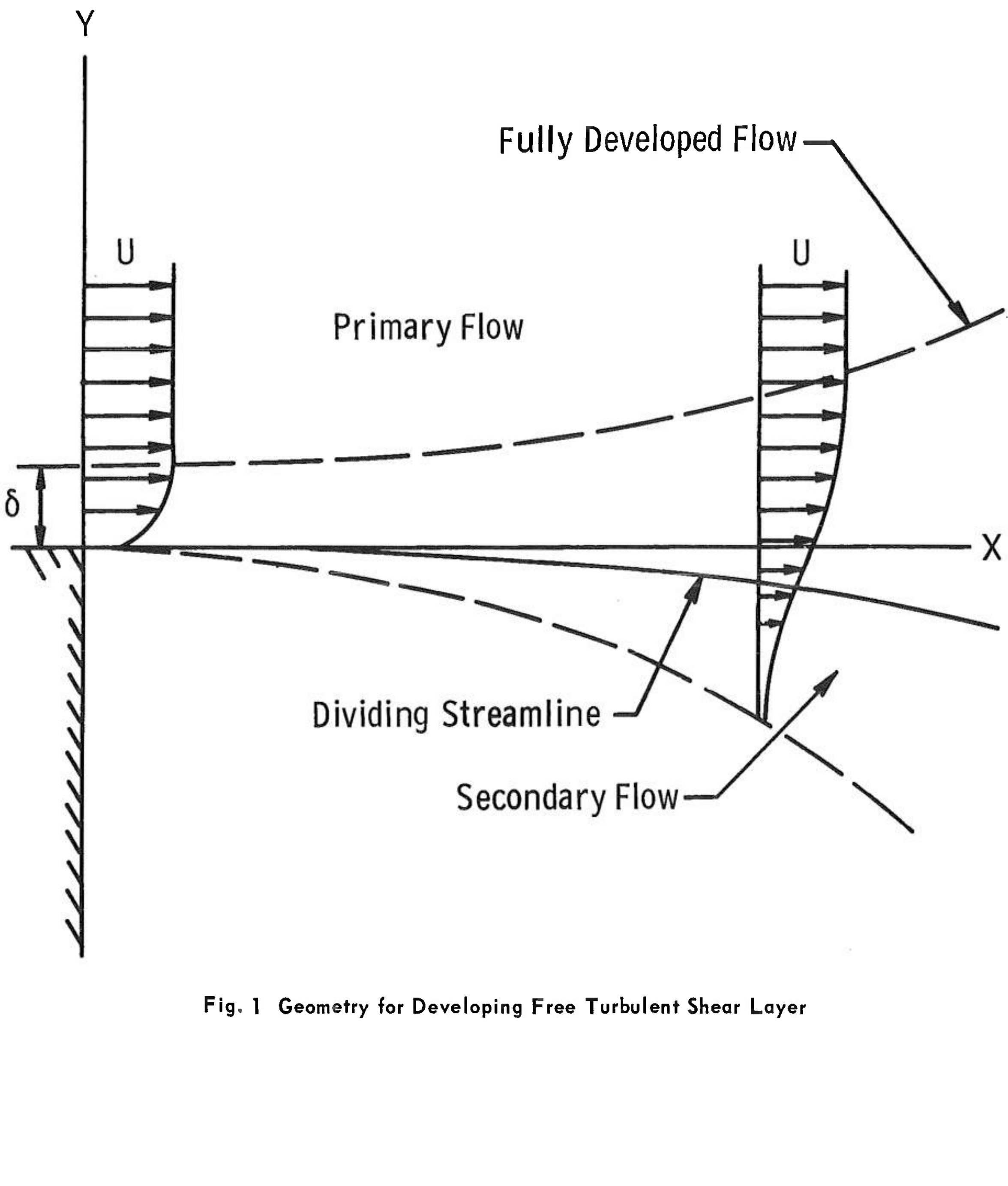


Fig. 1 Geometry for Developing Free Turbulent Shear Layer

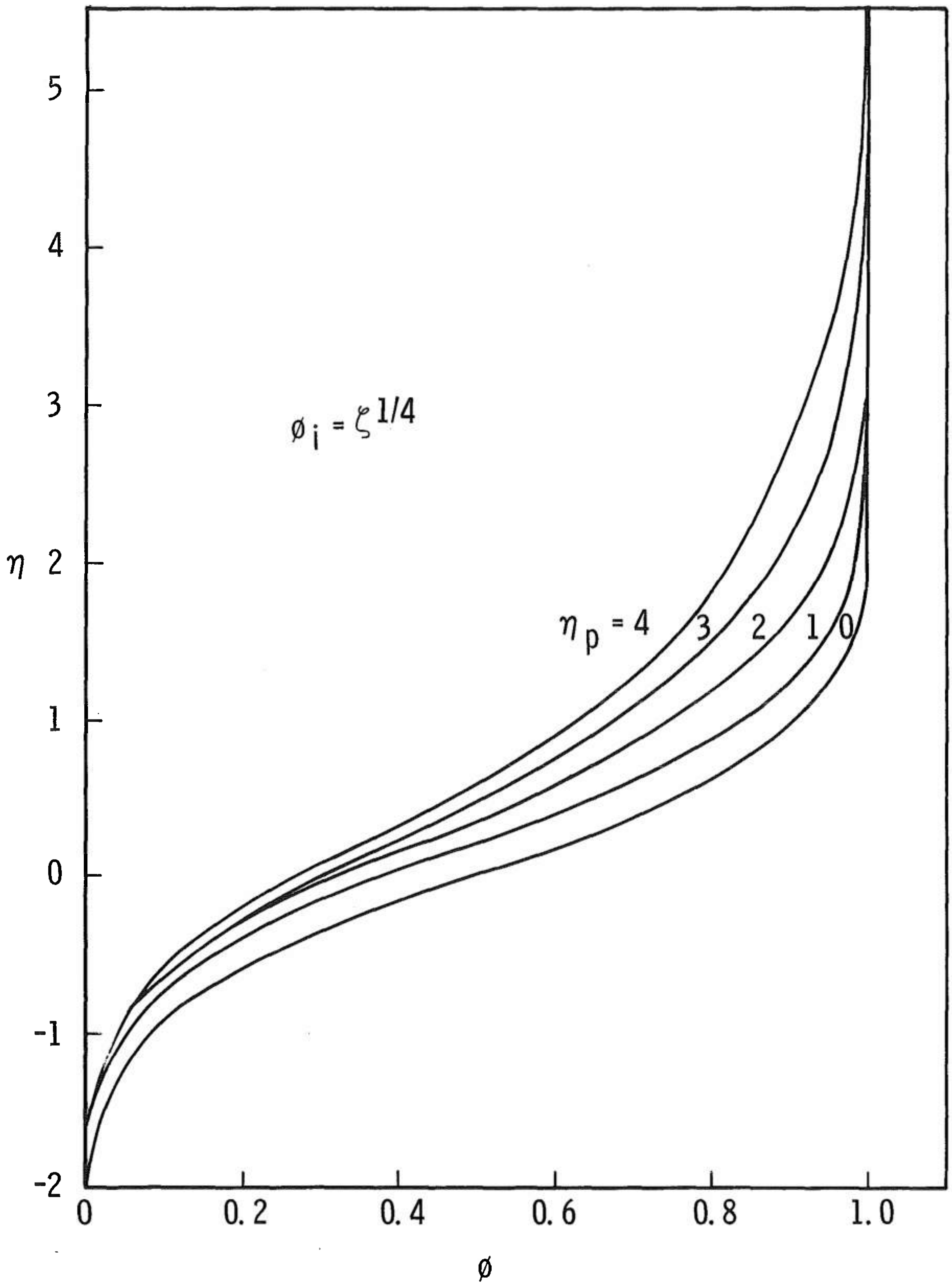


Fig. 2 Typical First-Order Velocity Profiles

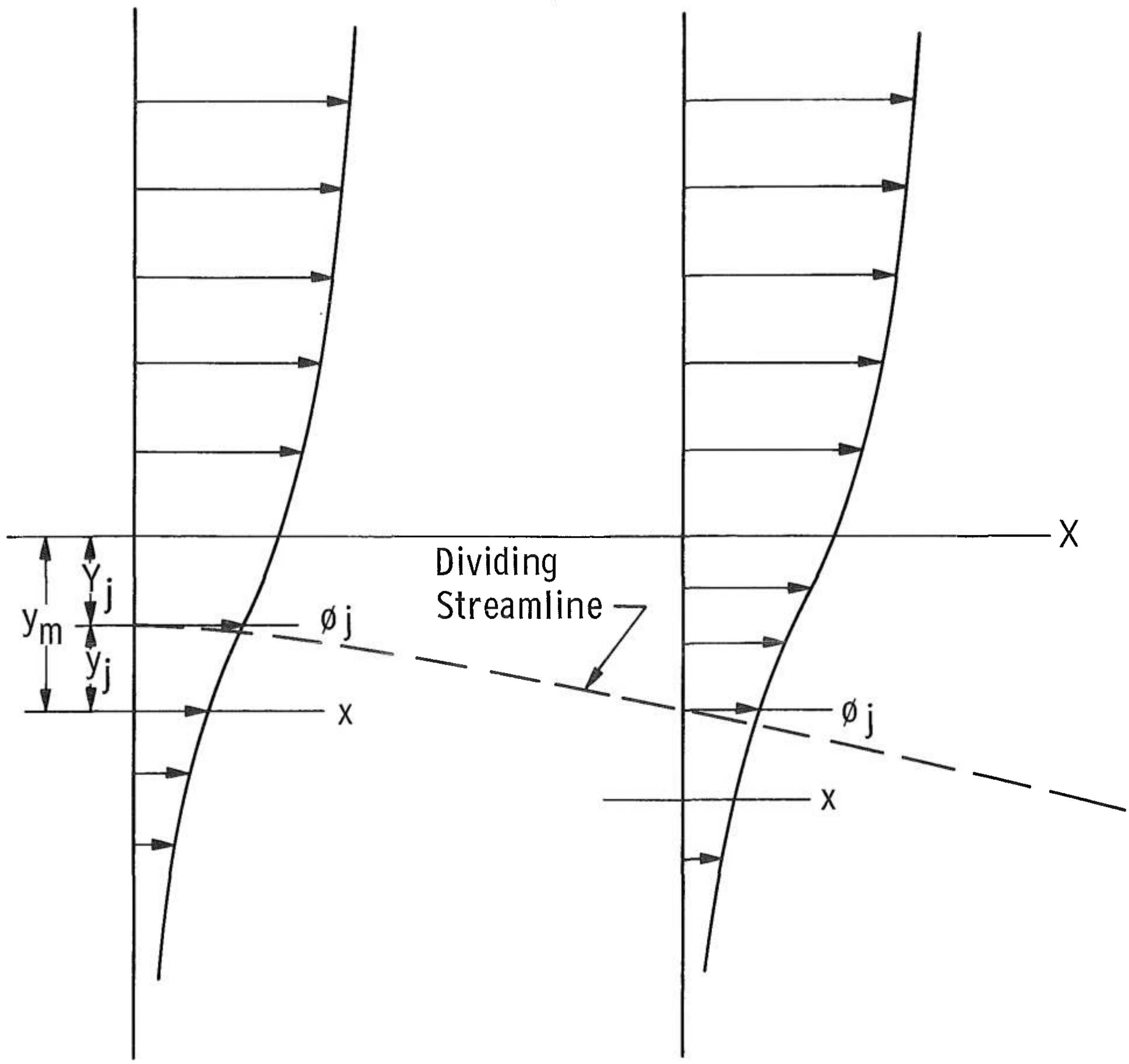
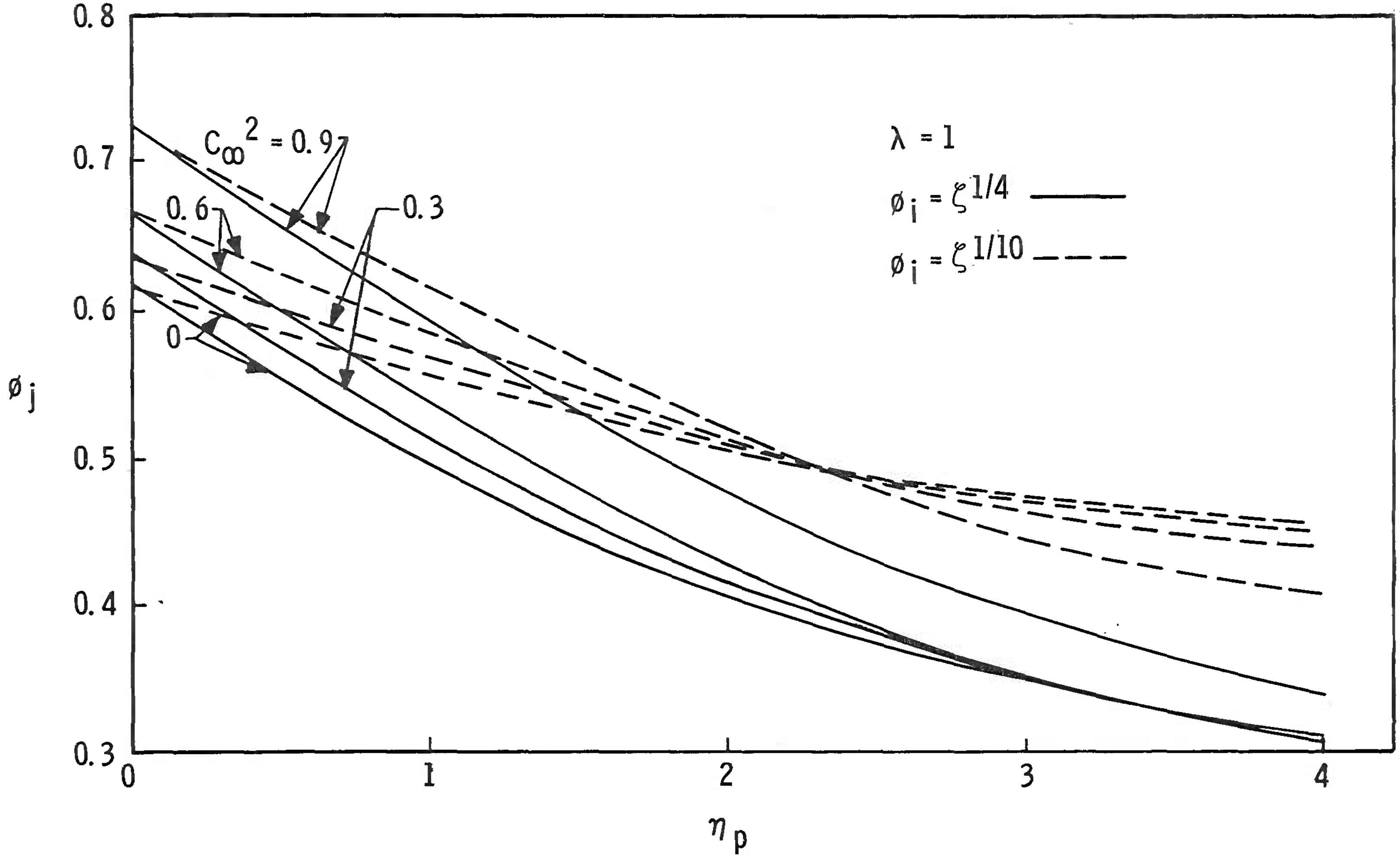
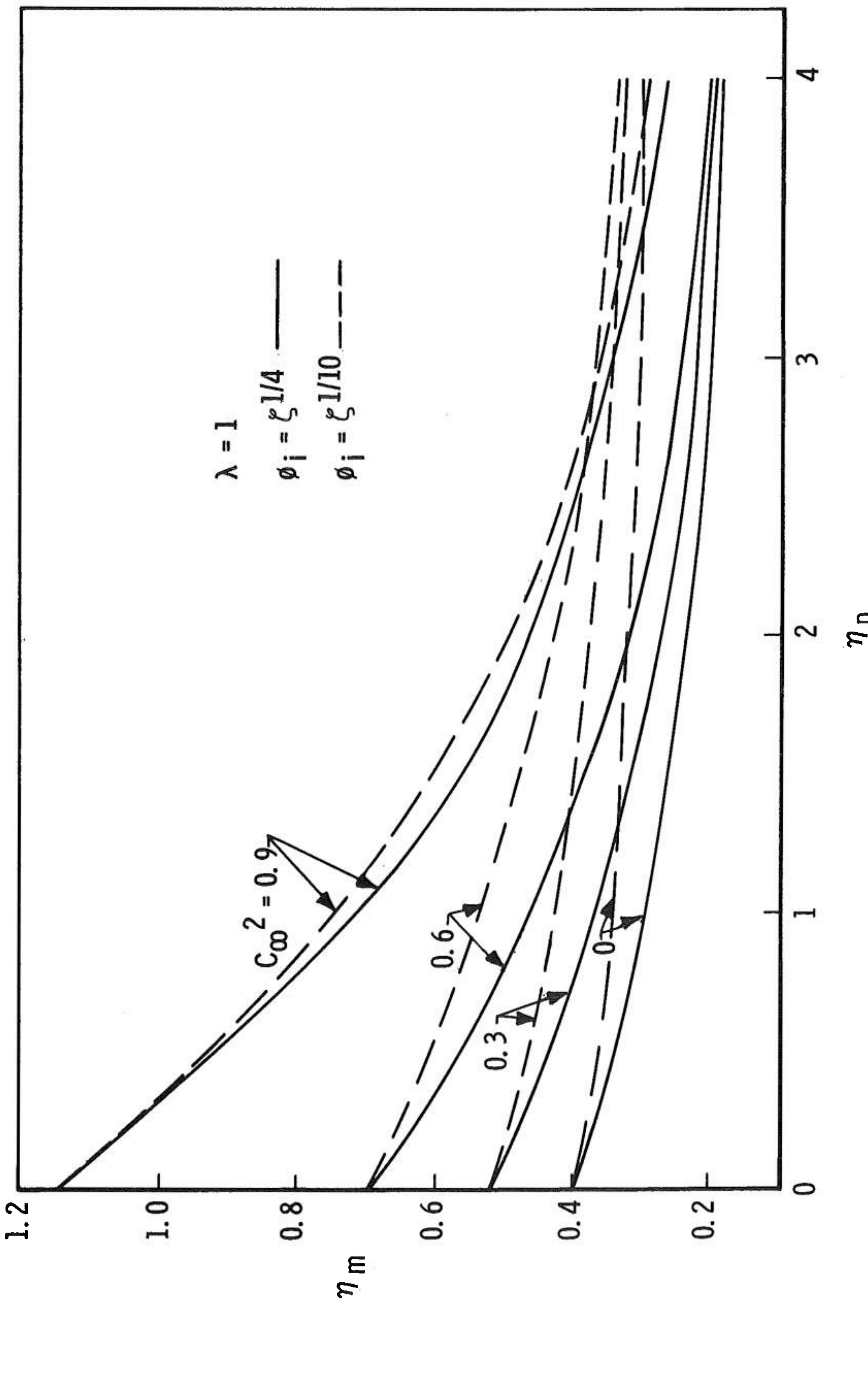


Fig. 3 Relation between Reference and Intrinsic Coordinates and the Dividing Streamline



a. Dividing Streamline Velocity

Fig. 4 Variation of Typical Mixing Zone Variables with Position Parameter



b. Coordinate Displacement Function

Fig. 4 Concluded

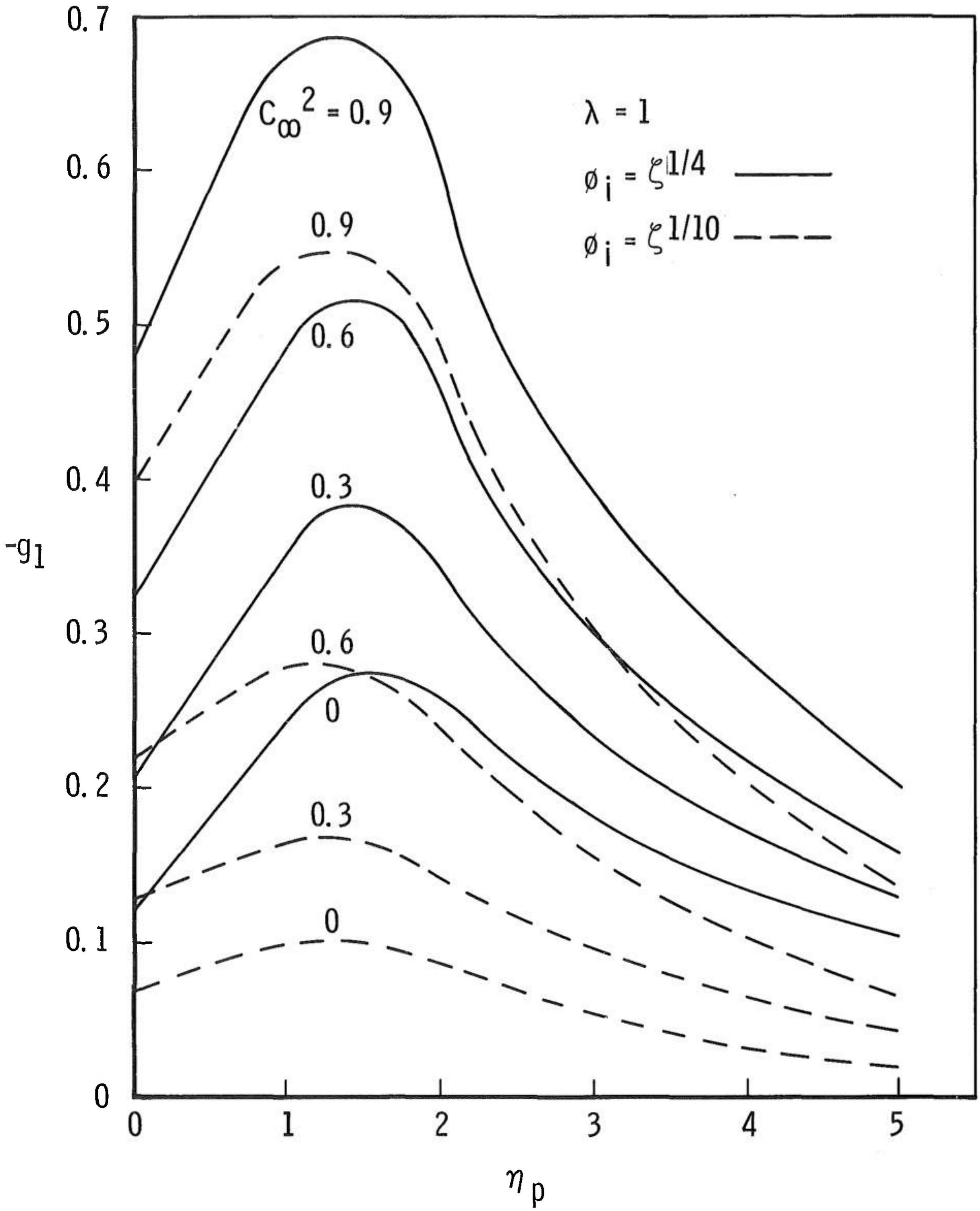


Fig. 5 Typical Variation of the Quantity, g_1

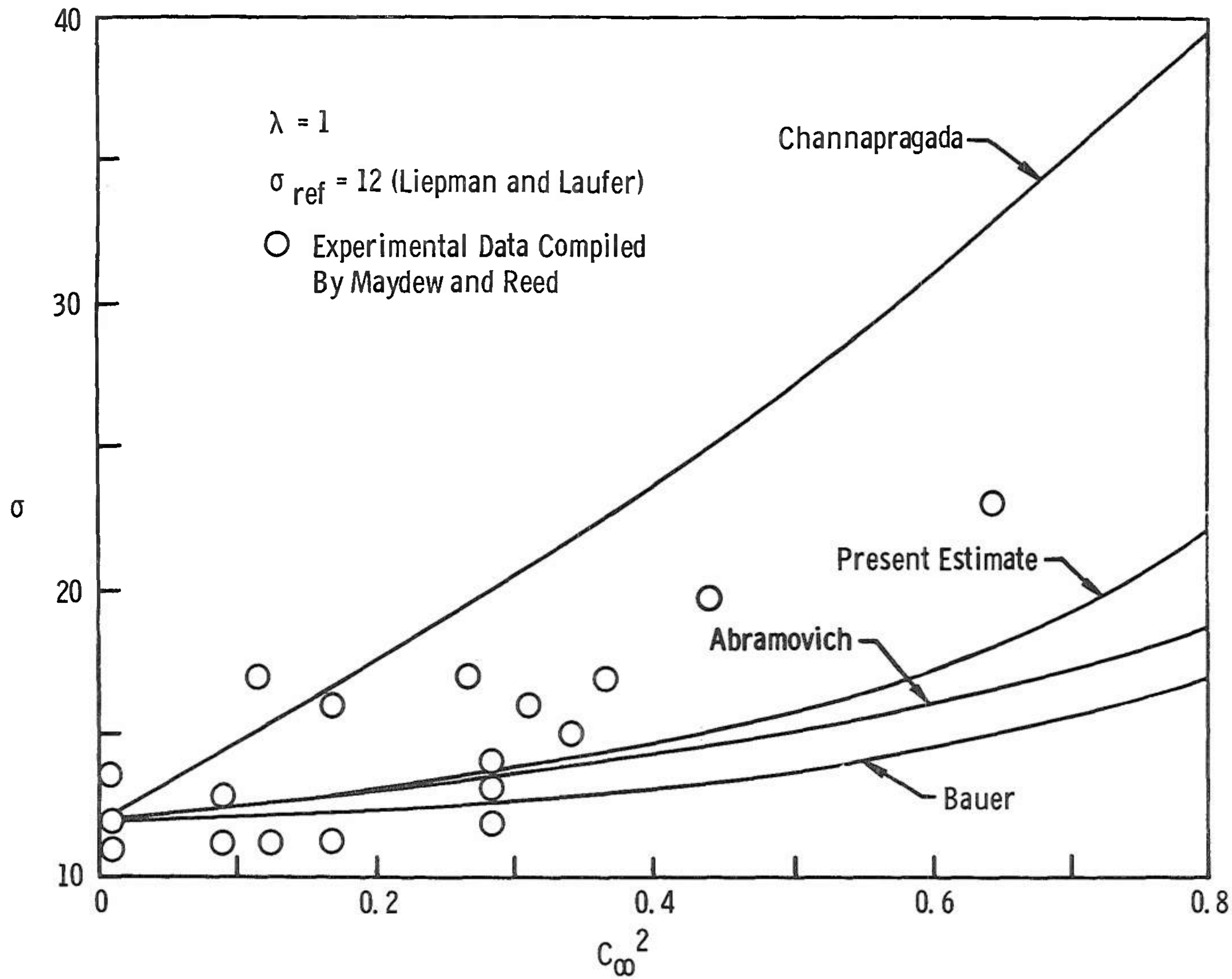


Fig. 6 Variation of the Asymptotic Spread Rate Parameter, σ, with Crocco Number for Isoenergetic Flow

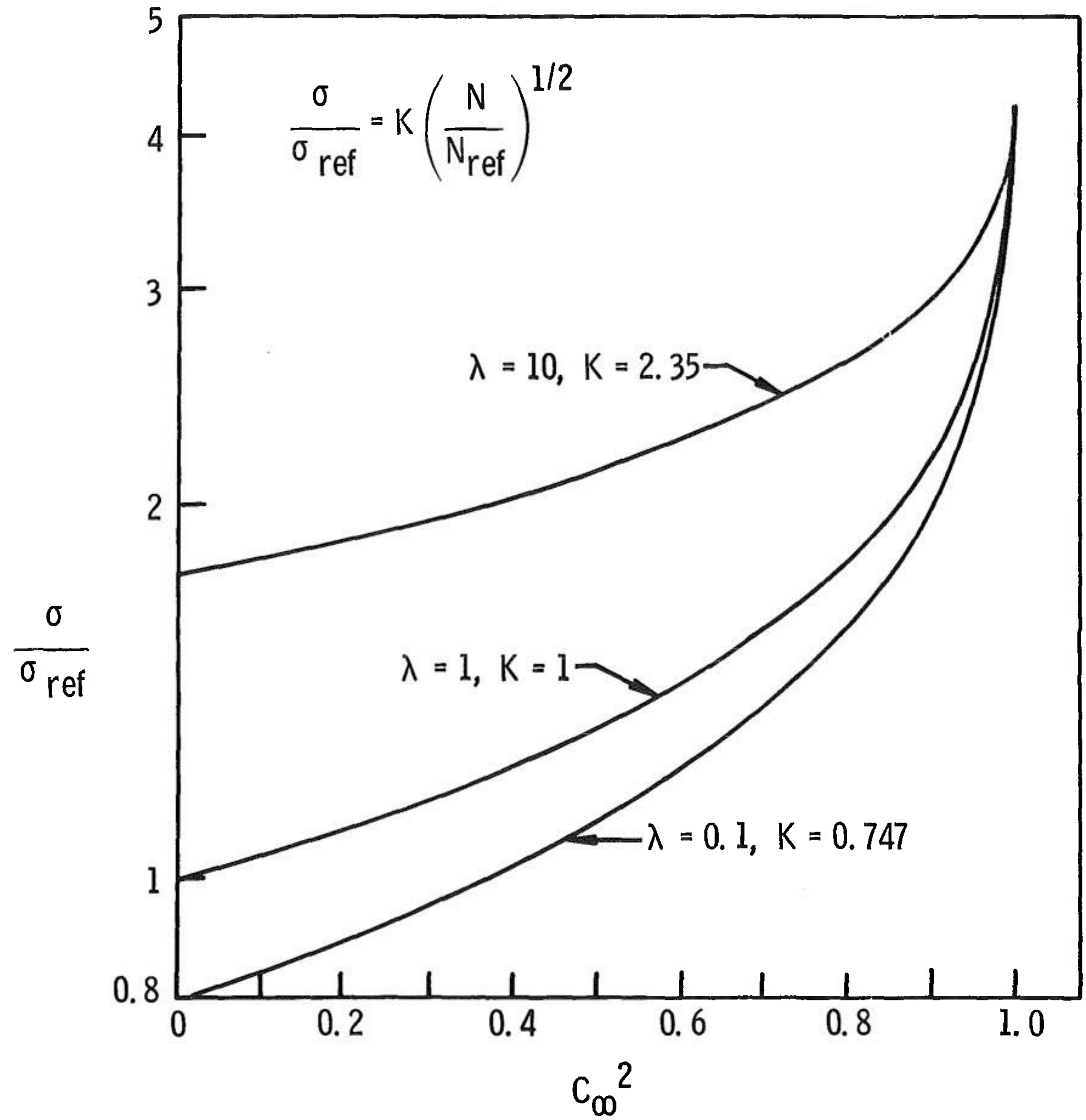


Fig. 7 Variation of σ/σ_{ref} with C_∞^2 and λ as Determined from Present Theory

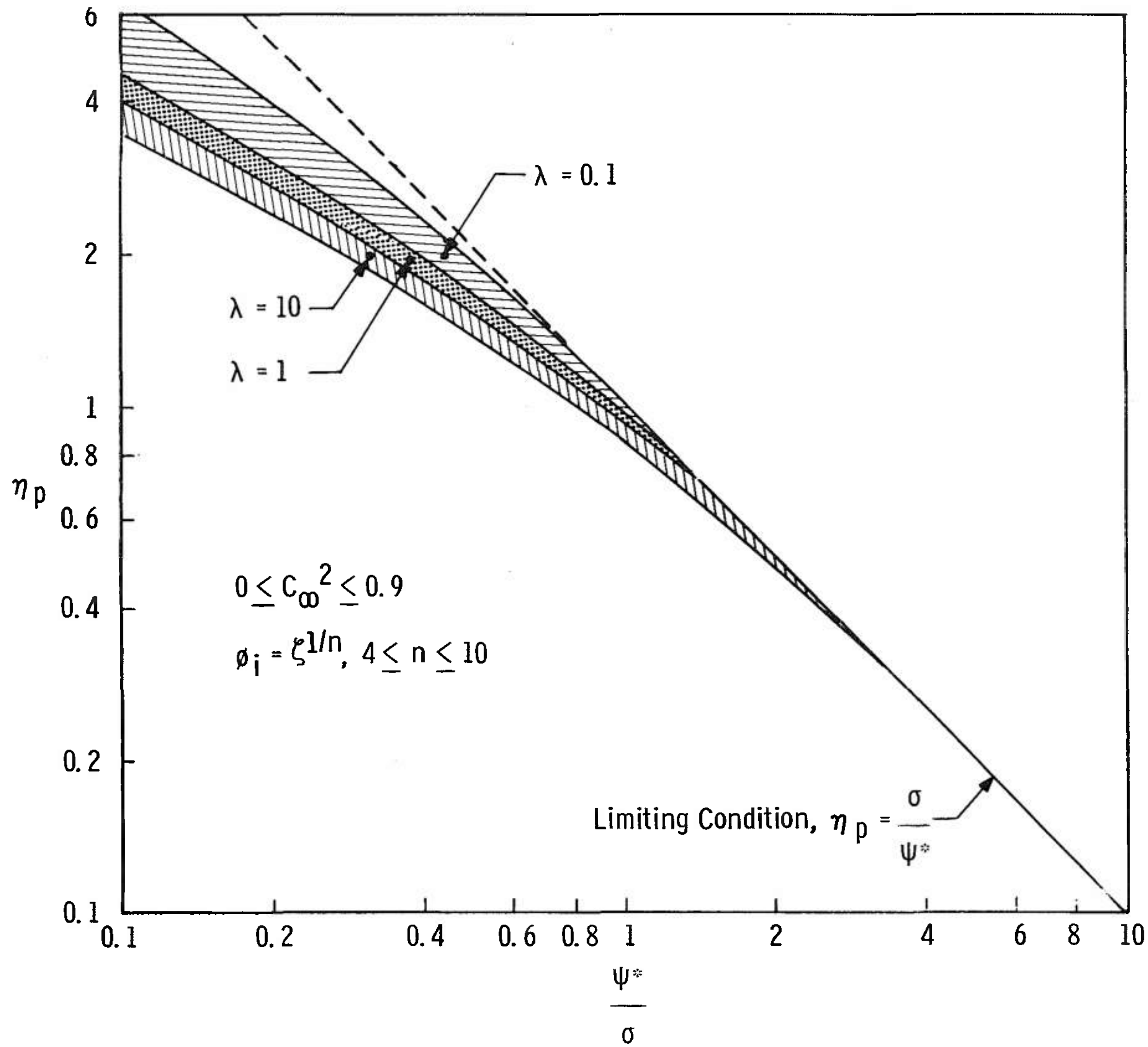


Fig. 8 Variation of the Position Parameter with Reduced Development Distance

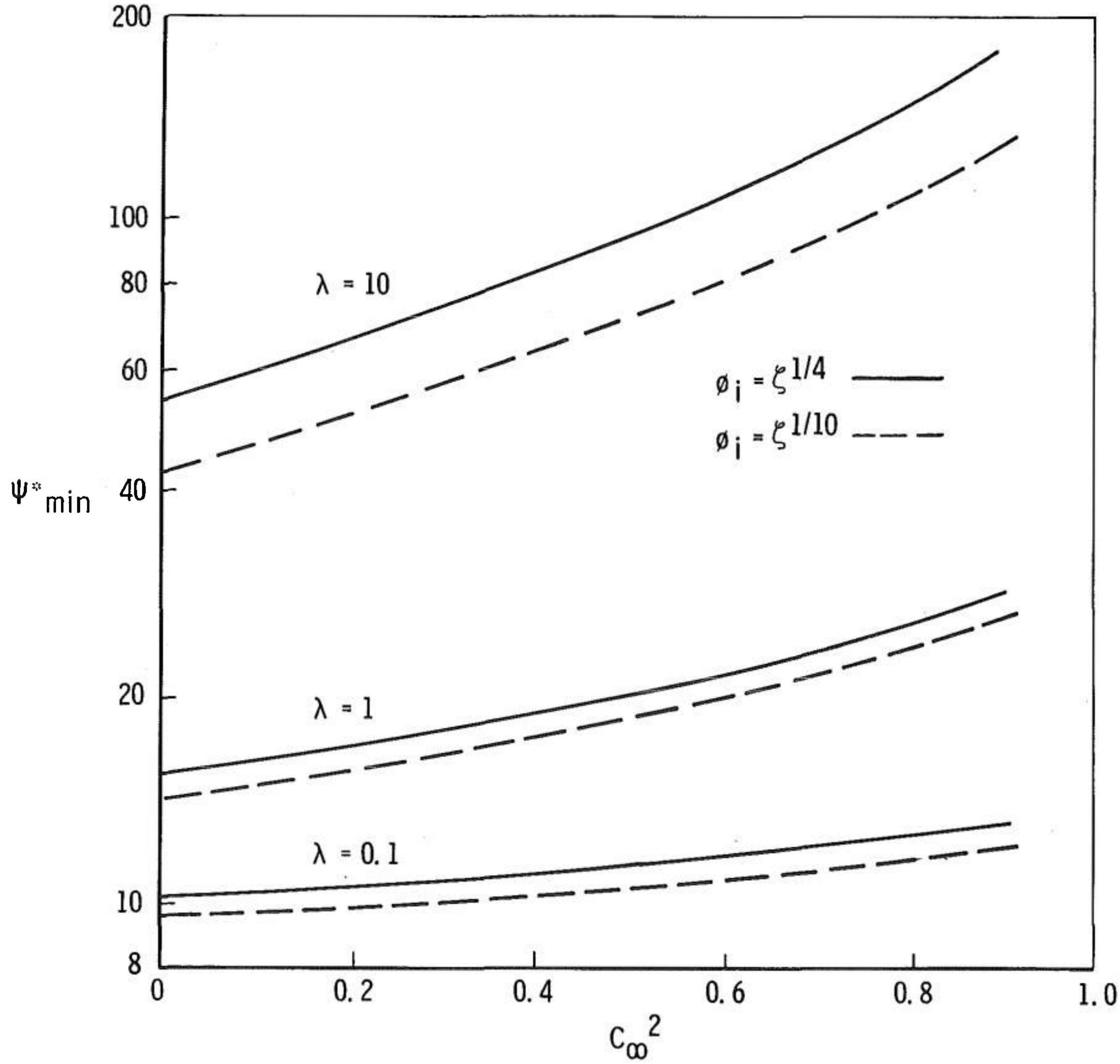


Fig. 9 Minimum Development Distance beyond which the Position Parameter Is Given by $\eta_p = \sigma/\Psi^*$

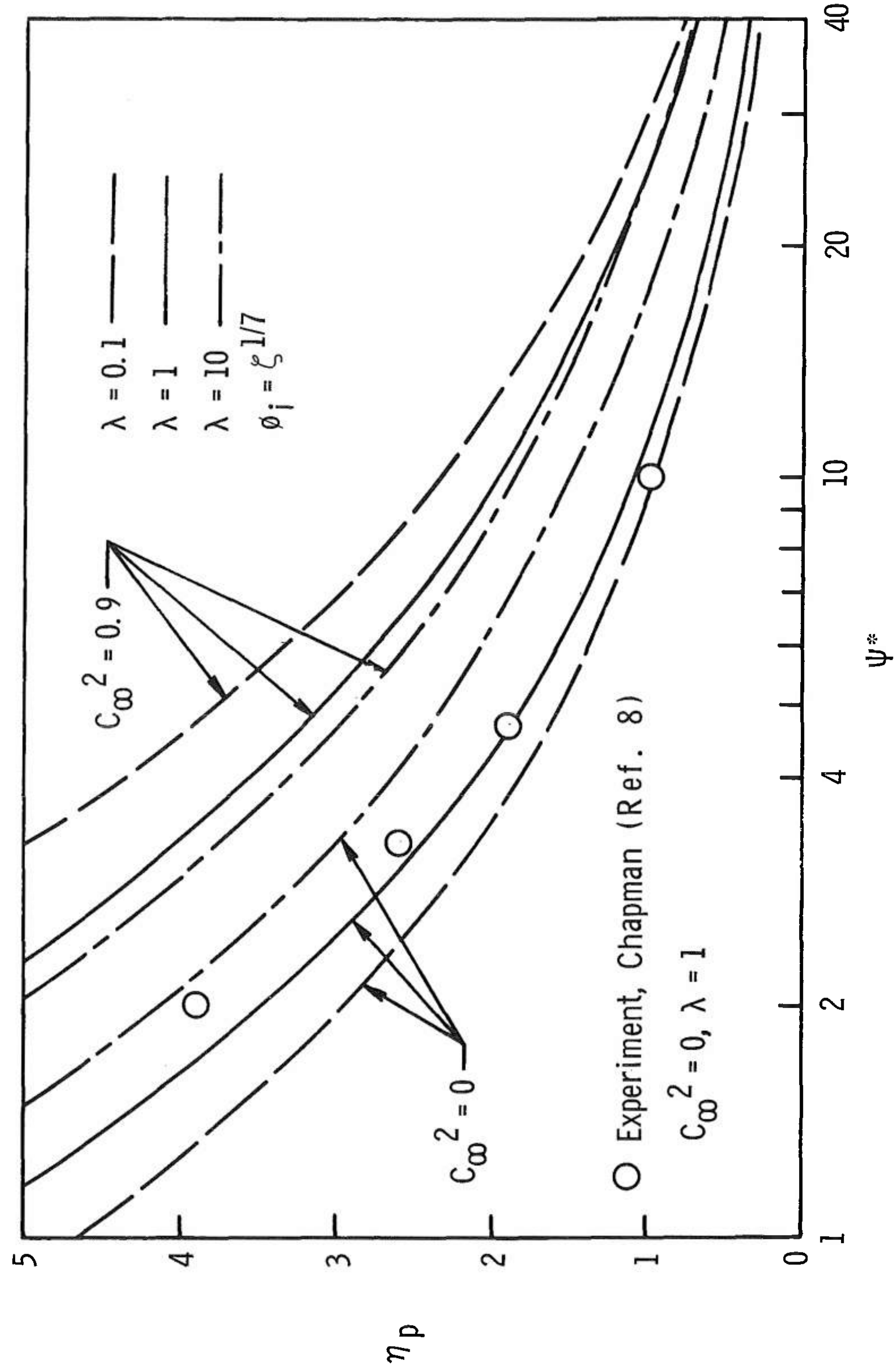


Fig. 10 Variation of Position Parameter with Actual Development Distance

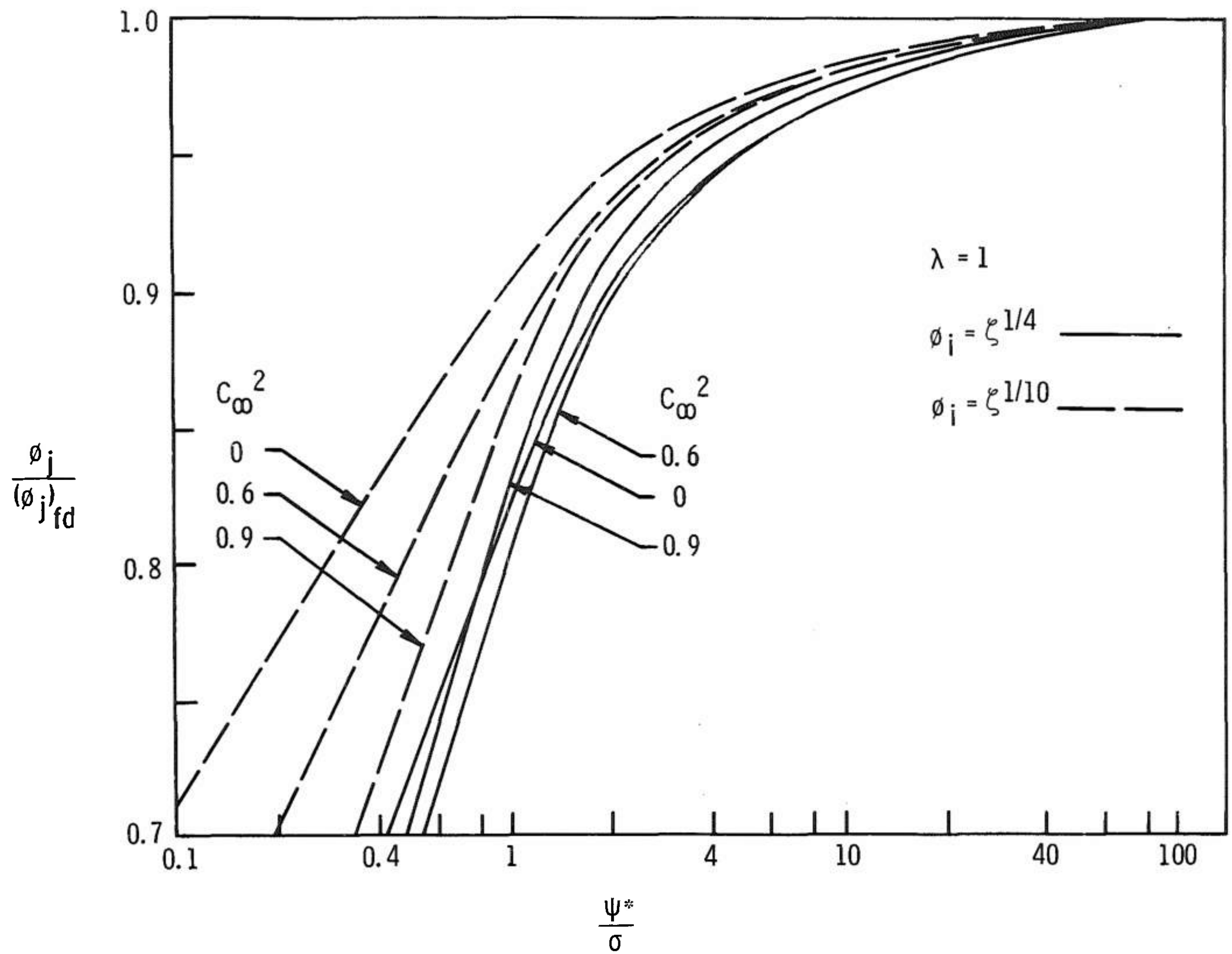


Fig. 11 Variation of Dividing Streamline Velocity with Reduced Development Distance

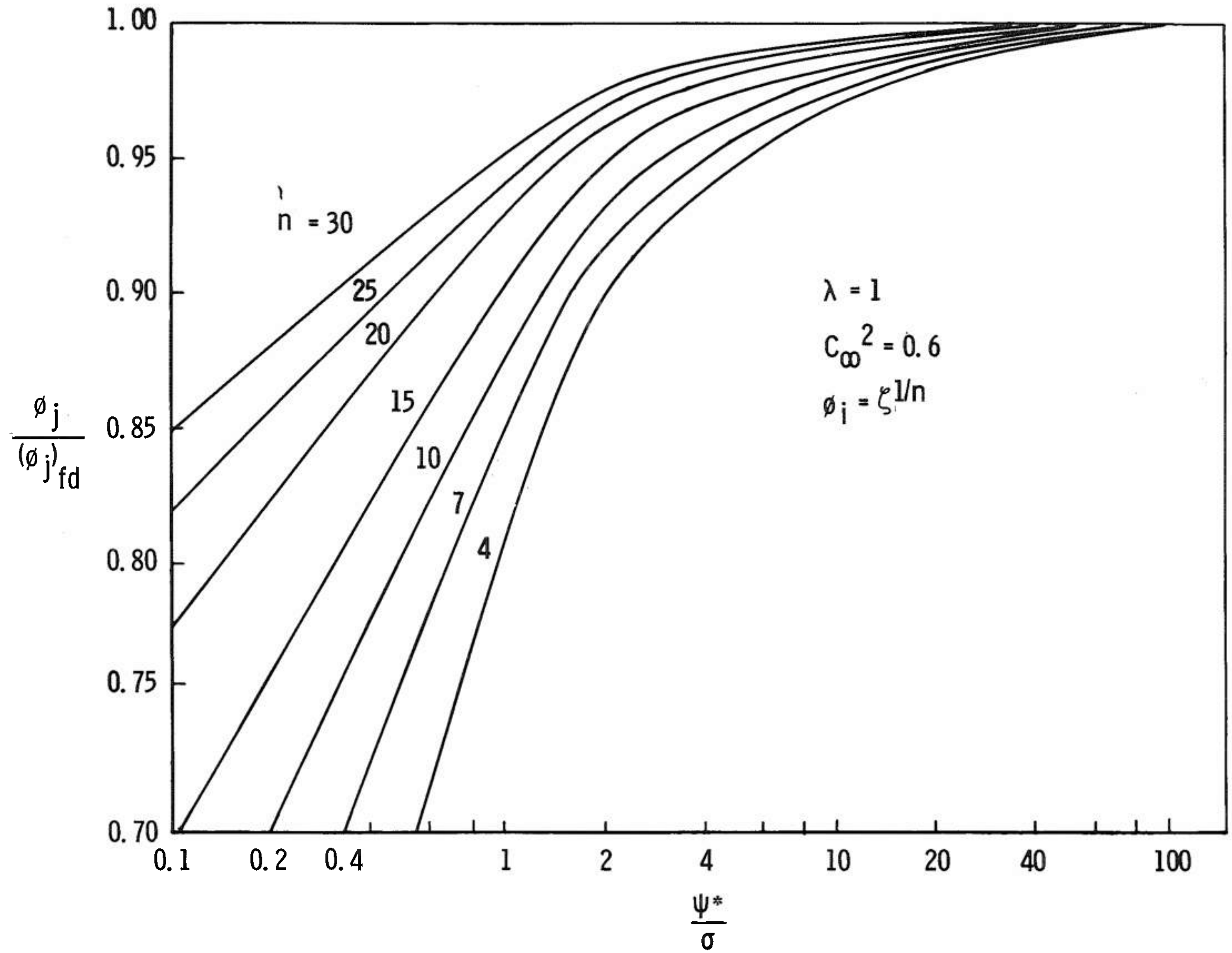


Fig. 12 Effect of Initial Velocity Profile on the Development of Dividing Streamline Velocity

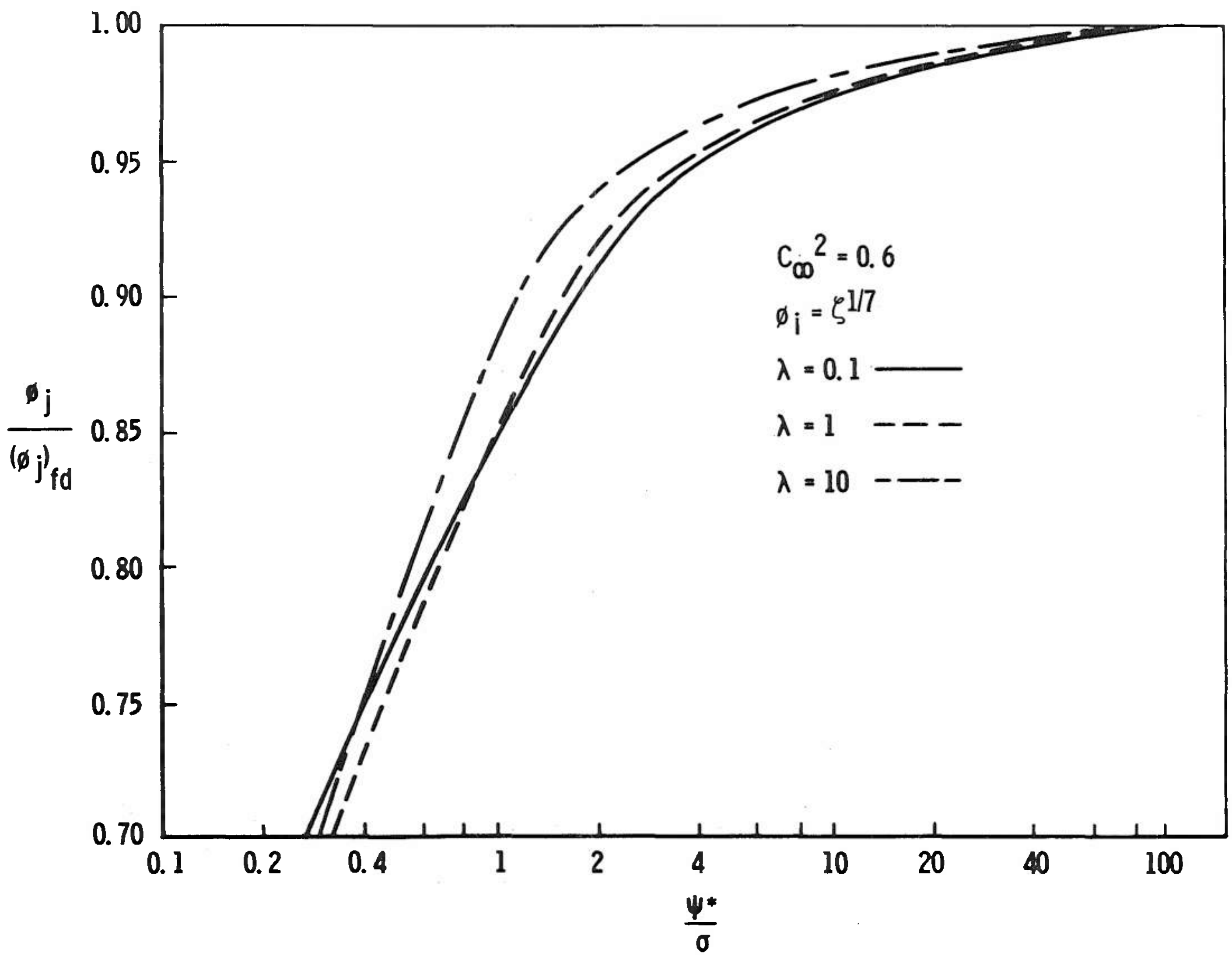


Fig. 13 Effect of Heat Transfer on the Development of the Dividing Streamline Velocity

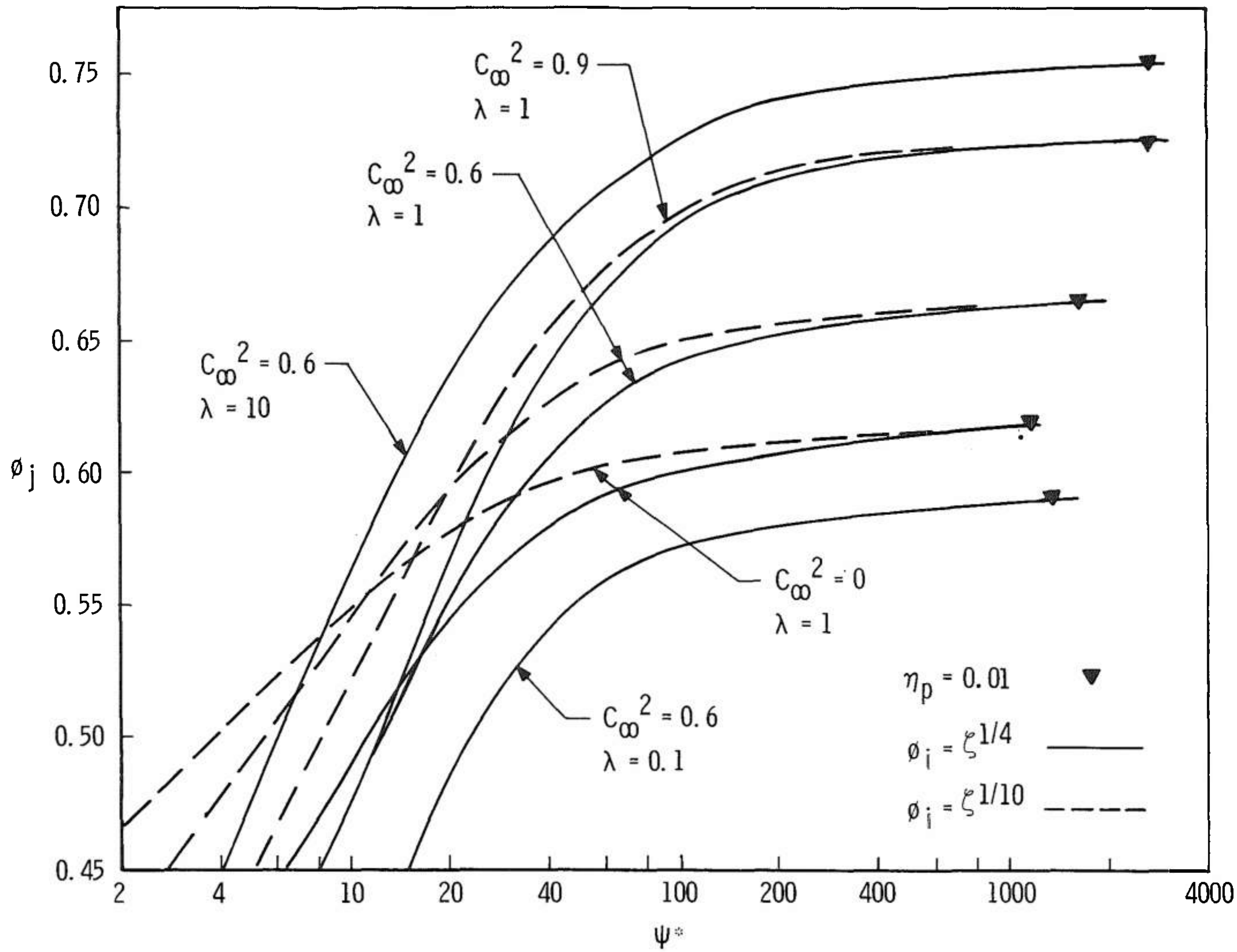


Fig. 14 Variation of Dividing Streamline Velocity with Actual Development Distance

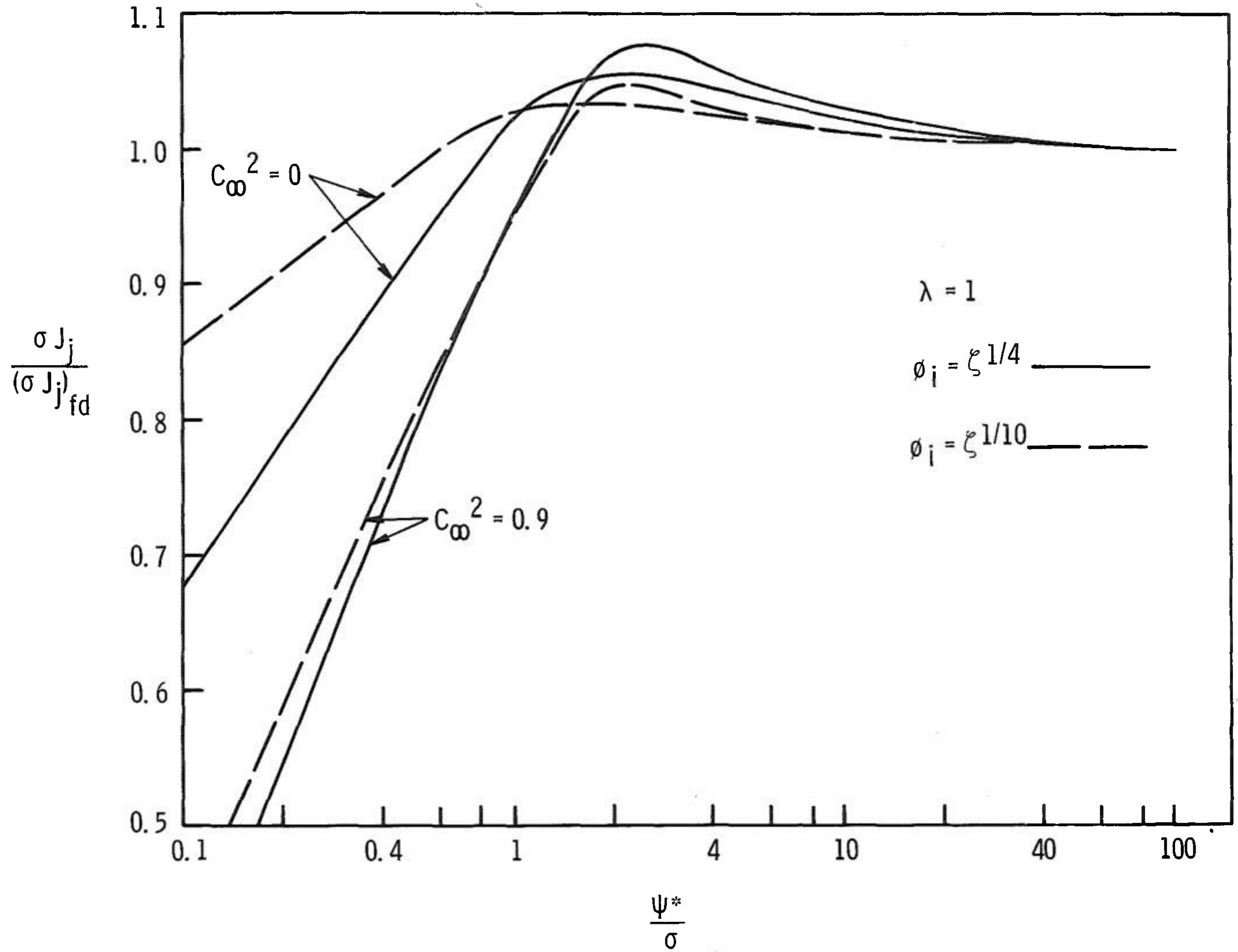


Fig. 15 Variation of Dividing Streamline Reduced Shear Stress with Reduced Development Distance for Isoenergetic Flow

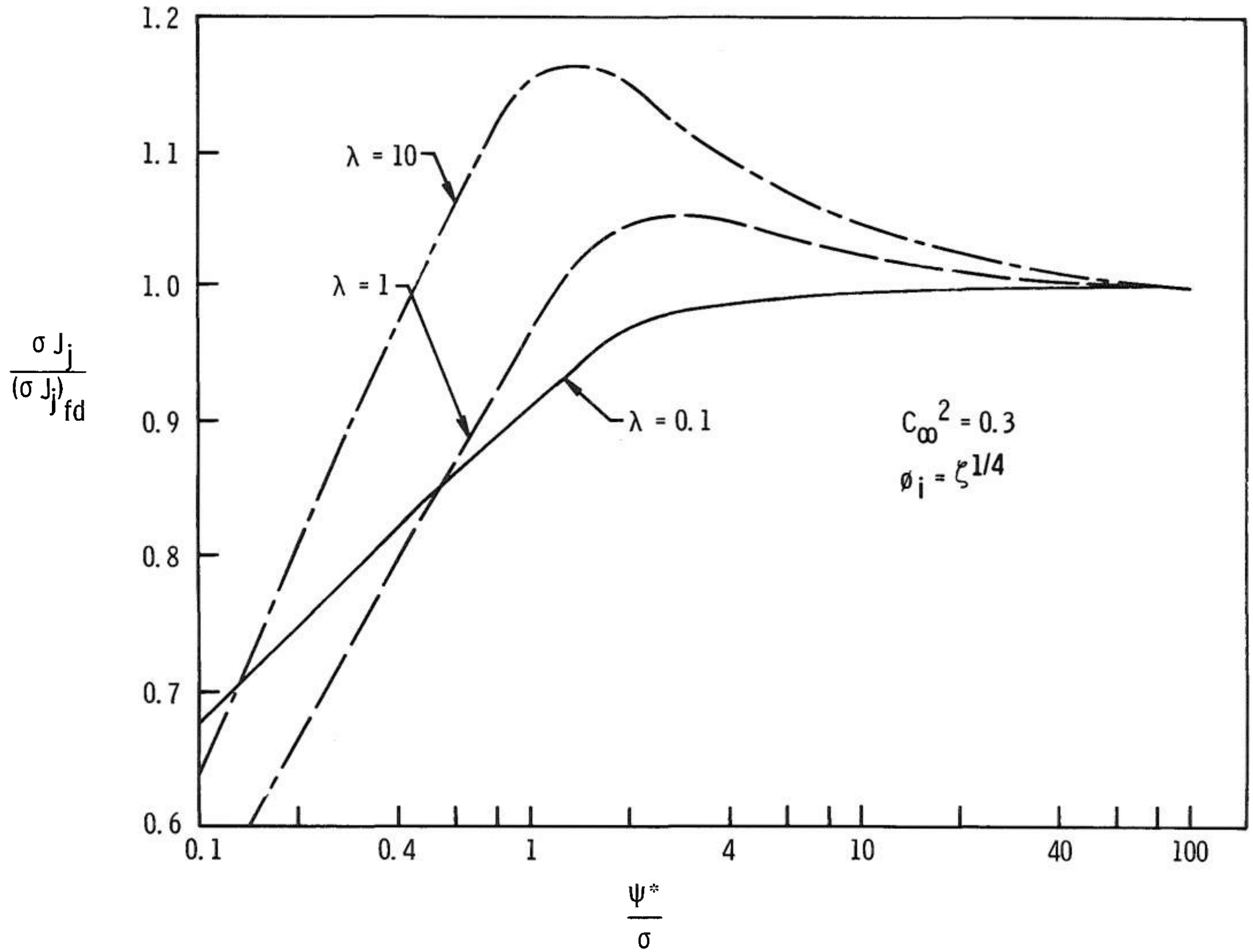


Fig. 16 Effect of Heat Transfer on the Development of the Dividing Streamline Reduced Shear Stress

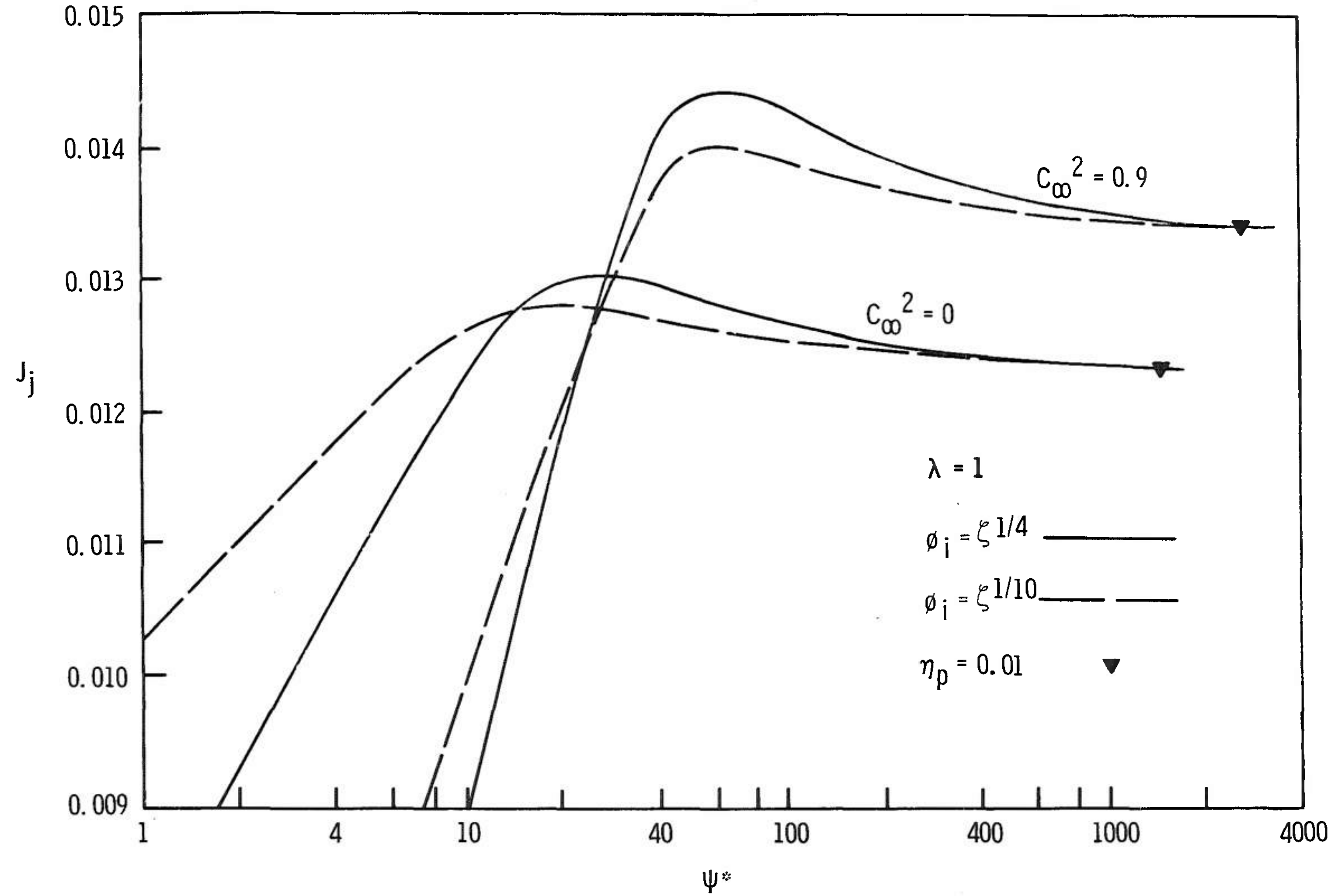


Fig. 17 Variation of Dividing Streamline Shear Stress with Actual Development Distance

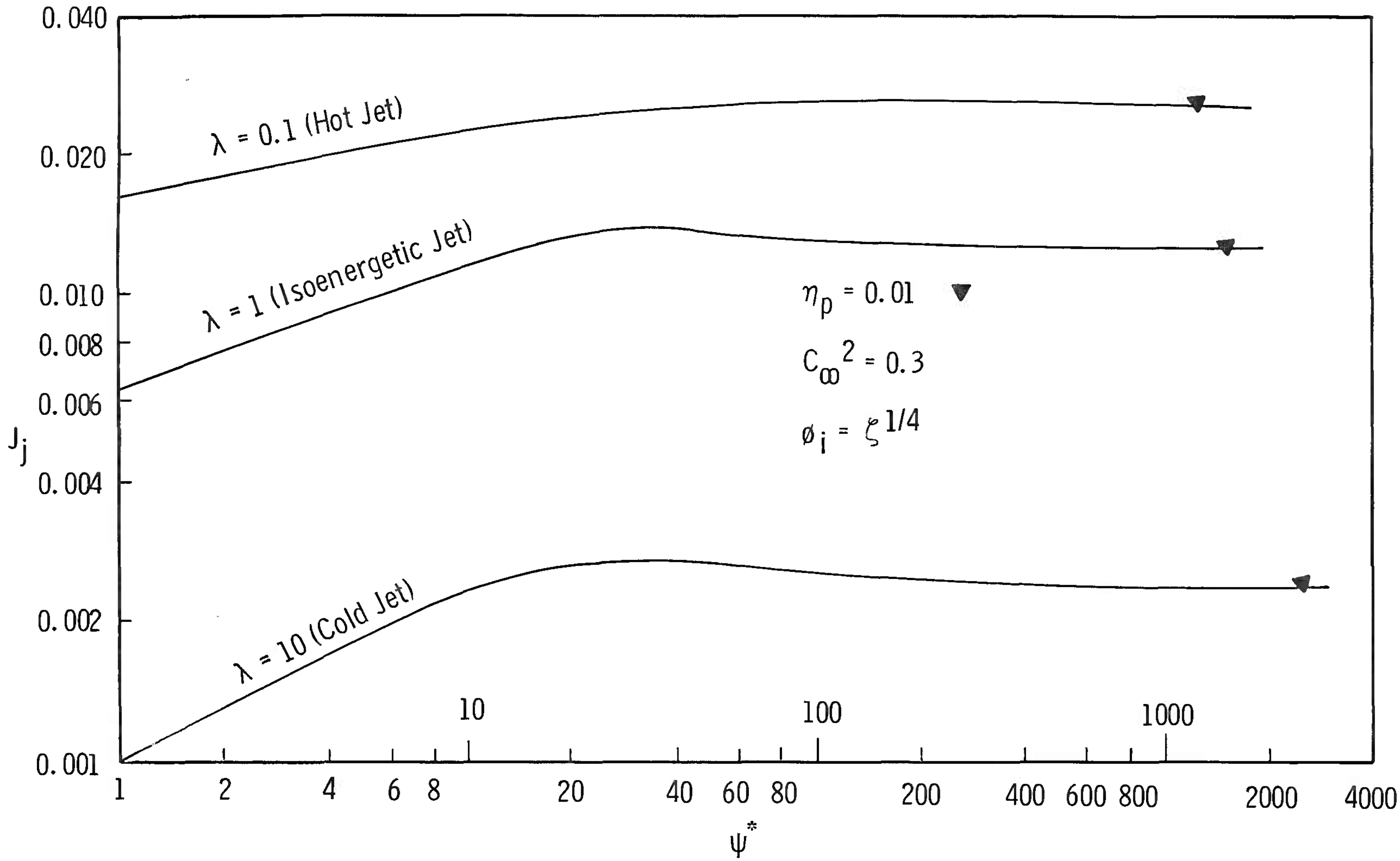


Fig. 18 Effect of Heat Transfer on Dividing Streamline Shear Stress

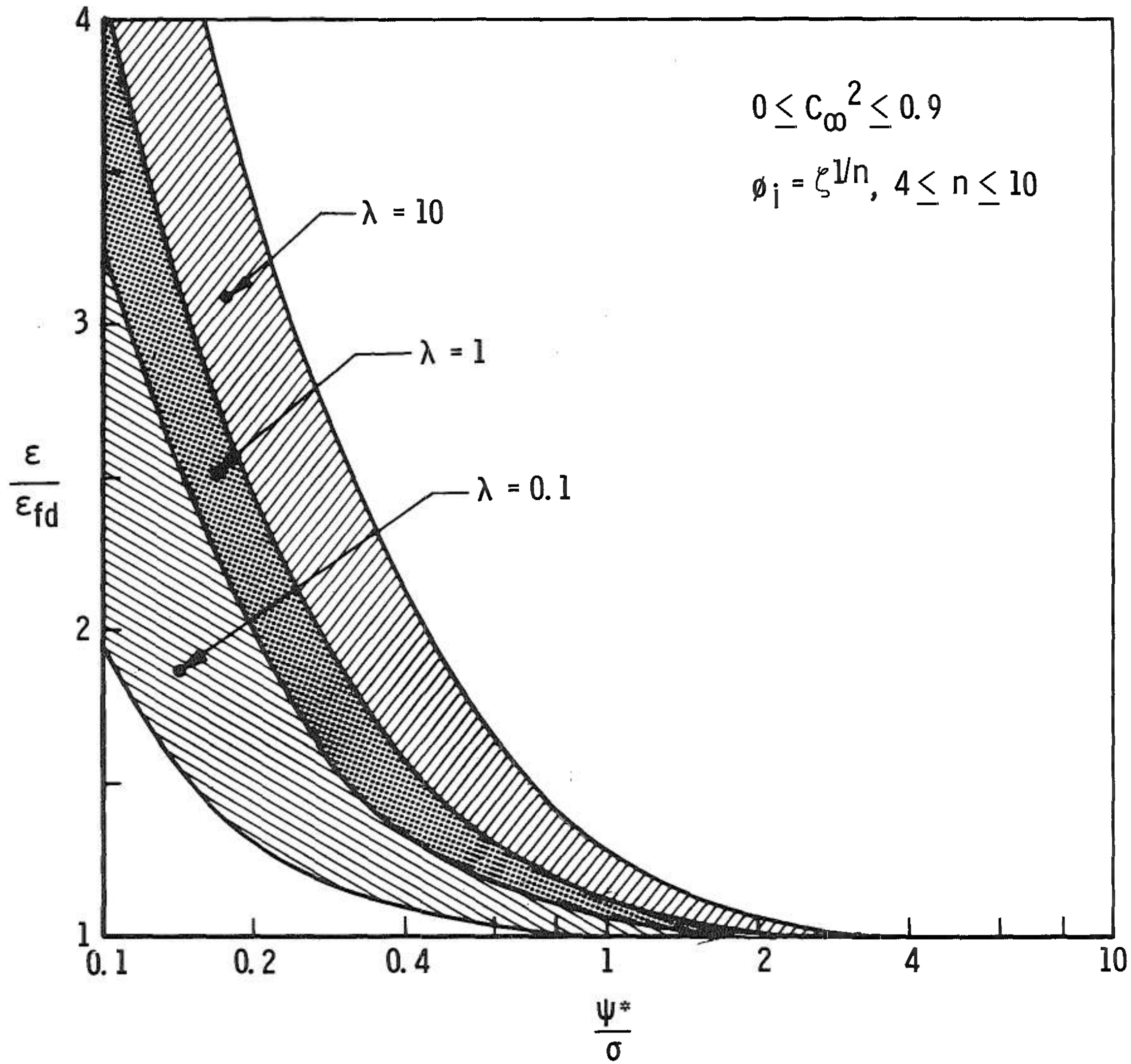


Fig. 19 Variation of Average Eddy Viscosity with Reduced Development Distance

TABLE I
VALUES OF σ/σ_{ref} DETERMINED FROM PRESENT THEORY

C_{∞}^2	Hot Jet, $\lambda = 0.1$	Isoenergetic Jet, $\lambda = 1$	Cold Jet, $\lambda = 10$
0	0.80	1.00	1.76
0.3	0.95	1.16	1.95
0.6	1.22	1.43	2.26
0.9	1.96	2.19	2.97
1.0	4.25	4.25	4.25

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13. ABSTRACT

The development of a two-dimensional, free turbulent shear layer from an arbitrary initial velocity profile is analyzed theoretically. Included in the analysis are effects of both heat transfer and compressibility. The mean flow is described by approximate velocity profiles containing an unknown position parameter which is dependent upon the development length. Integral forms of the continuity and momentum equations are utilized to specify the flow characteristics along the streamline which separates the primary and secondary flow regions. By applying the Navier-Stokes equations to this dividing streamline, one is able to calculate the position parameter and thus complete the description of the developing flow field. Also presented are results of extensive calculations which show, for various external and initial flow conditions, the variation of the dividing streamline velocity, shear stress (Stanton number), and average eddy viscosity. The theory also enables one to estimate the effects of heat transfer and compressibility on the spread rate parameter for fully developed mixing zones. The theoretical results are shown to agree with experimental data from a number of sources.

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